www.mtg.in | July 2015 | Pages 92 | ₹ 30

2015 SOLVED PAPERS

JEE ADVANCED | • KCET • COMEDK

JEE ACCELERATED LEARNING SERIES

PHYSICS for you

India's #1 PHYSICS MONTHLY FOR JEE (Main & Advanced) & PMTs

Thought Provoking Problems

PHYSICS Musing

Brain Map

CORE CONCEPT

You Ask ? We Answer ✓

ACE YOUR WAY CBSE XII

SERIES 2

PHYSICS for you

Vol. XXIII No. 7 July 2015

Corporate Office :
Plot 99, Sector 44 Institutional area,
Gurgaon -122 003 (HR). Tel : 0124-4951200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat (BE, MBA)

Contents

Owned, Printed and Published by Mahabir Singh from 406, Taj Apartment, New Delhi - 29 and printed by Personal Graphics and Advertisers (P) Ltd., Okhla Industrial Area, Phase-II, New Delhi. Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.
Editor : Anil Ahlawat

editorial

The Pilgrim's Progress in Physics

From astronomy to the atom bomb, Physics has passed through many stages. First, physical observations and measurements of time by means of pendulums were made and related to the theory. From the Newtonian era, theory was formulated from experiments and then extrapolations were made from theory to further refine the experiments.

Studies on astronomical events were going hand in hand. The transition of geometrical optics to physical optics gave the twin concepts of particle and wave for light where light was supposed to exist both as a wave as well as a particle. But the nature of the particle as well as the wave was unknown. The theory of Relativity of Einstein enabled the results of Compton effect to be explained. The fundamental basis of the classical physics was the constancy of the length, mass and time. The theory of relativity showed that the length, mass and time are not constants but dependent on the velocity of the body. The only constant used by Einstein was the velocity of light in vacuum. Photons, the particles of light as well as the material particles such as electrons, protons and neutrons have a double aspect as particles and waves.

The twentieth century physics was galloping with the theoretical physicists being in command. This was purely a progress caused by the meditation of the great minds. We have in India, great scientists and thinkers. We advise the students to learn from them and add your own contribution to the world.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

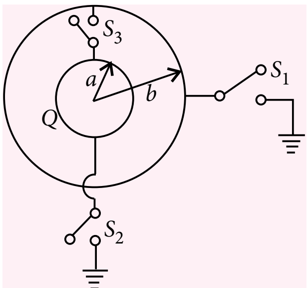
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 24

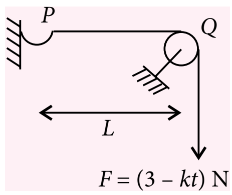
SUBJECTIVE TYPE

1. A large concave mirror with a radius of curvature of 0.5 m is placed on a horizontal plate. We pour a small amount of water into the mirror and place on it a plano-convex lens whose curved surface has radius of curvature of 1.0 m pointing upwards and plane side just touches the water surface. Then we place a point light source above this set-up at a height of 0.3 m. At what distance (in metres) from the object does the final image form? [The refractive index of water is 1.33 and the refractive index of glass is 1.50]

2. The figure shows a conducting sphere A of radius a which is surrounded by a neutral conducting spherical shell B of radius b ($> a$). Initially switches S_1, S_2 and S_3 are open and sphere A carries a charge Q. First the switch S_1 is closed to connect the shell B with the ground and then opened. Now the switch S_2 is closed so that the sphere A is grounded and then S_2 is opened. Finally, the switch S_3 is closed to connect the spheres together. Find the heat (in joule) which is produced after closing the switch S_3. [Consider a = 2 cm , b = 4 cm and Q = 8 μC]

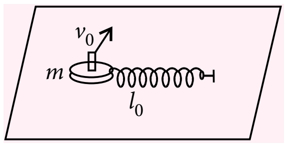


3. In the given figure, a string of linear mass density 3×10^{-2} kg m^{-1} and length L = 1 m, is stretched by a force $F = (3 - kt)$ N, where k is a constant and t is time in second. At the time t = 0, a pulse is generated at the end P of the string. Find the value of k (in N s^{-1}) if the value of force becomes zero as the pulse reaches point Q.



4. A solid cylinder of mass m = 6 kg and radius r = 0.1 m is kept in balance on a slope of inclination α = 37° with the help of a thread fastened to its jacket. The cylinder does not slip on the slope. What is the minimum required co-efficient of friction to keep the cylinder in balance when the thread is held vertically ?

5. One side of a spring of initial, unstretched length l_0 = 1 m lying on a frictionless table, is fixed, the other one is fastened to a small puck of mass m = 0.1 kg. The puck is given velocity in a direction perpendicular to the spring, at an initial speed v_0 = 11 m s^{-1}. In the course of the motion, the maximum elongation of the spring is $l = \frac{l_0}{10}$. What is the force constant of the spring (in SI units) ?

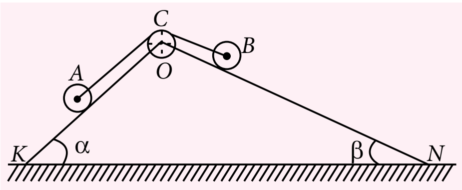


6. A Bohr hydrogen atom undergoes a transition $n = 5 \rightarrow n = 4$ and emits a photon of frequency υ. Frequency of circular motion of electron in n = 4 orbit is υ_4. Find the ratio $\frac{\upsilon}{\upsilon_4}$.

7. A wheel A is connected to a second wheel B by means of inextensible string, passing over a pulley C, which rotates about a fixed horizontal axle O, as shown in figure. The system is released from rest. The wheel A rolls down the inclined plane OK thus pulling up the wheel B which rolls along the inclined plane ON. Determine the velocity (in m s^{-1}) of the axle of the wheel A, when it has travelled a distance

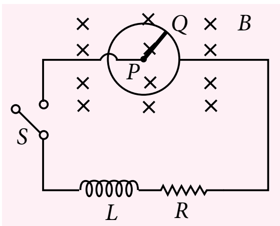
By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Senior Professor Physics, RAO IIT ACADEMY, Mumbai.

$s = 3.5$ m down the slope. Both wheels and the pulley are assumed to be homogeneous disks of identical weight and radius. Neglect the weight of the string. [Take : $g = 10$ m s^{-2}, $\alpha = 53°$ and $\beta = 37°$]

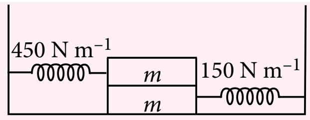


8. One mole of an ideal monoatomic gas is taken from state A to state B through the process $P = \frac{3}{2}T^{1/2}$. It is found that its temperature increases by 100 K in this process. Now it is taken from state B to C through a process for which internal energy is related to volume as $U = \frac{1}{2}V^{1/2}$. Find the total work performed by the gas (in joule) , if it is given that volume at B is 100 m^3 and at C it is 1600 m^3. [Use $R = 8.3$ J mol^{-1} K^{-1}]

9. The diagram shows a circuit having a coil of resistance $R = 10\ \Omega$ and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its centre at P. Assume that friction and gravity are absent and a constant uniform magnetic field of 5 T exists as shown in figure. At $t = 0$, the circuit is switched on and simultaneously a time varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad s^{-1}. Find magnitude of this torque (in milli N m) when current reaches half of its maximum value. Neglect the self inductance of the loop formed by the circuit.



10. When the system shown in the diagram is in equilibrium, the right spring is stretched by 1 cm . The co-efficient of static friction between the blocks is 0.3. There is no friction between the bottom block and the supporting surface. The force constants of the springs are 150 N m^{-1} and 450 N m^{-1} (refer figure). The blocks have equal mass of 2 kg each. Find the maximum amplitude (in cm) of the oscillations of the system shown in the figure that does not allow the top block to slide on the bottom.



SOLUTION OF JUNE 2015 CROSSWORD

[1]W				[2]F																			
[3]A	B	[4]S	O	R	P	T	A	N	C	E													
V		T		[5]A	L	B	E	D	O				[6]C		[7]L				[8]M		[9]B		
E		E		U							[10]Z		O		Y		[11]Q	U	A	[12]S	A	R	
P		A		N							E		L		M				G	T	R		
U		M		[13]H	A	[14]W	K	I	N	G	R	A	D	I	A	T	[15]I	O	N	A	Y		
L		[16]P	H	O	N	O	N				O		F		N		C		E	T	O	[17]T	
S		O		F		R		[18]P			P		U		S		E		T	I	N	H	
E		I		E		M		L			O		S		E		P		O	C		E	
		N		R		H		E			I		I		R		O		L	S		R	
		T				O		N			N		O		I		I		O			M	
						L		I			T		N		E		N		[19]G	L	U	O	N
[20]M	U	L	T	I	V	E	R	S	E		E				S		T		Y			P	
A								M			N			[21]M	I	L	L	I	B	A	R	I	
G									[22]P	N	E	U	M	A	T	I	C	S				L	
N											R			[23]S	C	R	E	W				E	
[24]O	E	R	S	T	E	D					G												
X				[25]B	O	U	N	D	A	R	Y			[26]R	E	S	O	N	A	N	C	E	

WINNERS (June 2015)

- **Dibyakanti Kumar**
- **Sunil Kumar Rath (Odisha) :** *"Crossword helped me a lot to revise. I am very much grateful to MTG group for organizing crossword competition in every issue of Physics for You".*
- **Shreya Sharma (New Delhi)**

Solution Senders (May 2015)

- **Shambhavi Chowdhary (Rajasthan) :** *"A regular MTG cross-word dose keeps away all exam woes!!!."*
- **Gawali Aishwarya Balasaheb (Maharashtra)**
- **Harsh S. Hathi (Gujrat) :** *"This crossword is amazing from the point of view of knowledge and very helpful for further studies. The level of question is excellent."*

Solution Senders of Physics Musing

SET-23

1. Swarnendu Bhattacharjee (WB)
2. Swayangdipta Bera (WB)
3. Anurag Banerjee (New Delhi)
4. Sanjay Singh (Haryana)
5. Sayanta Bhanja (WB)

SET-22

1. Swarnendu Bhattacharje (WB)
2. Anju Dhall (Haryana)

JEE Advanced

SOLVED PAPER 2015

PAPER-1

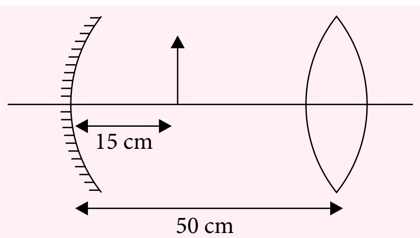
SECTION 1 (Maximum Marks : 32)

- *This section contains EIGHT questions*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive*
- *For each question, darken the bubble corresponding to the correct integer in the ORS*
- *Marking scheme:*

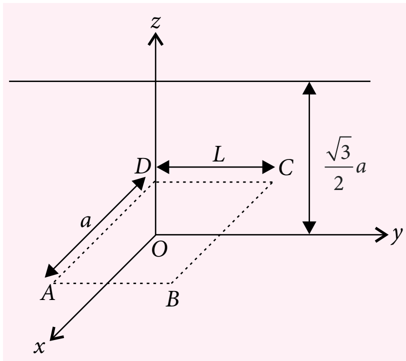
  +4 If the bubble corresponding to the answer is darkened

  0 In all other cases

1. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



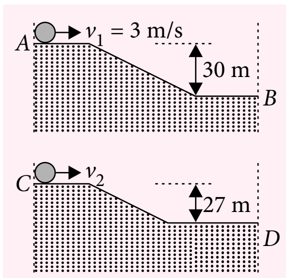
2. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



3. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm)

4. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{\text{th}}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

5. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces.

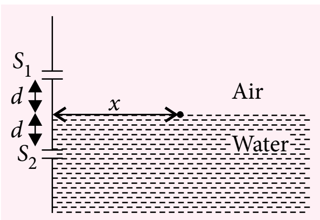


If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s^2)

6. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

7. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

8. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is



SECTION 2 (Maximum Marks : 40)

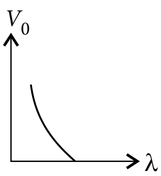
- *This section contains TEN questions*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS*
- *Marking scheme :*

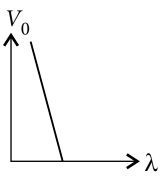
  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

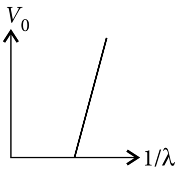
  0 If none of the bubbles is darkened

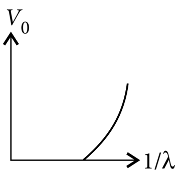
  –2 In all other cases

9. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $1/\lambda$.

(a) 

(b) 

(c) 

(d) 

10. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then

(a) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(b) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(c) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

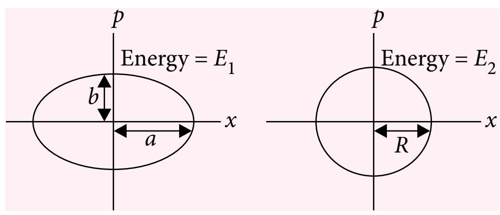
(d) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

11. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is(are)

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$

(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

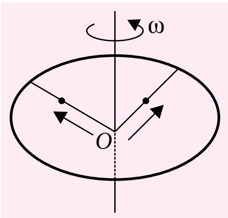
12. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are)



(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

13. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is

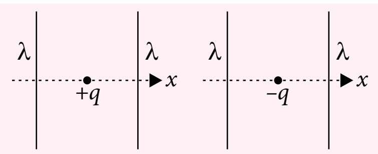


(a) $\frac{2}{3}R$

(b) $\frac{1}{3}R$

(c) $\frac{3}{5}R$ (d) $\frac{4}{5}R$

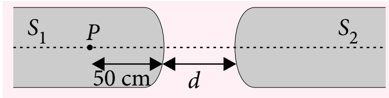
14. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is(are)



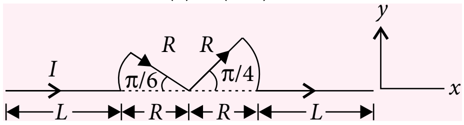
(a) Both charges execute simple harmonic motion.
(b) Both charges will continue moving in the direction of their displacement.
(c) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.
(d) Charge $-q$ executes simple harmonic motion while charge $+q$ continues moving in the direction of its displacement.

15. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

(a) 60 cm
(b) 70 cm
(c) 80 cm
(d) 90 cm



16. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is(are)



(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$
(b) If $\vec{B}$ is along $\hat{x}$, $F = 0$
(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$
(d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

17. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are)

(a) The average energy per mole of the gas mixture is $2RT$.
(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.
(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.
(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

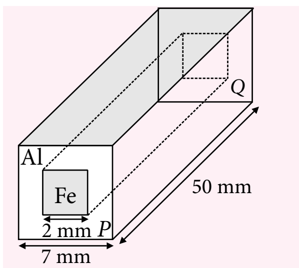
18. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\ \Omega$ m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

(a) $\frac{2475}{64}\ \mu\Omega$

(b) $\frac{1875}{64}\ \mu\Omega$

(c) $\frac{1875}{49}\ \mu\Omega$

(d) $\frac{2475}{132}\ \mu\Omega$



SECTION 3 (Maximum Marks : 16)

- *This section contains TWO questions*
- *Each question contains two columns, Column I and Column II*
- *Column I has four entries (A), (B), (C) and (D)*
- *Column II has five entries (P), (Q), (R), (S) and (T)*
- *Match the entries in Column I with the entries in Column II*
- *One or more entries in Column I may match with one or more entries in Column II*
- *The ORS contains a 4 5 matrix whose layout will be similar to the one shown below:*

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- *For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).*
- *Marking scheme:*
  For each entry in Column I
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened
  0 If none of the bubbles is darkened
  –1 In all other cases

19. Match the nuclear processes given in column I with the appropriate options(s) in column II.

Column I		Column II	
(A)	Nuclear fusion	(P)	Absorption of thermal neutrons by ${}^{235}_{92}U$
(B)	Fission in a nuclear reactor	(Q)	${}^{60}_{27}Co$ nucleus
(C)	β-decay	(R)	Energy production in stars via hydrogen conversion to helium
(D)	γ-ray emission	(S)	Heavy water
		(T)	Neutrino emission

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I		Column II	
(A)	$U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P)	The force acting on the particle is zero at $x = a$.
(B)	$U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q)	The force acting on the particle is zero at $x = 0$.
(C)	$U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R)	The force acting on the particle is zero at $x = -a$.
(D)	$U_4(x) = \frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S)	The particle experiences an attractive force towards $x = 0$ in the region $\|x\| < a$.
		(T)	The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

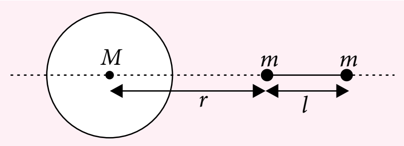
PAPER-2

SECTION 1 (Maximum Marks : 32)

- *This section contains EIGHT questions*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive*
- *For each question, darken the bubble corresponding to the correct integer in the ORS*
- *Marking scheme:*
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

1. An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\,\pi\,a_0$ (where a_0 is the Bohr radius). The value of p is

2. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



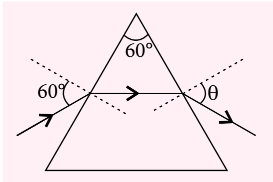
3. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ \text{s}^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of $E(t)$ at $t = 5$ s is

4. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

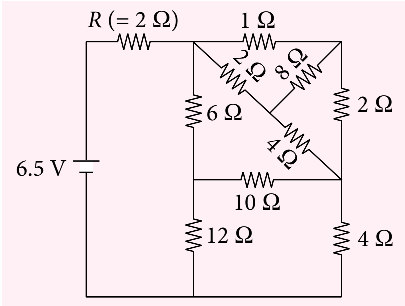
5. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

6. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

7. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



8. In the following circuit, the current through the resistor R (= 2 Ω) is I Amperes. The values of I is



SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT questions*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS*
- *Marking scheme :*
  - *+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened*
  - *0 If none of the bubbles is darkened*
  - *–2 In all other cases*

9. A fission reaction is given by

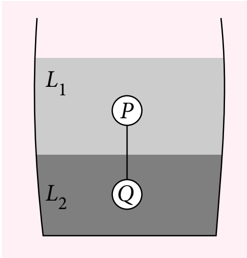
$$^{236}_{92}\text{U} \to {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y,$$

where x and y are two particles. Considering $^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x (2 MeV) and K_y (2 MeV), respectively. Let the binding energies per nucleon of $^{236}_{92}\text{U}$, $^{140}_{54}\text{Xe}$ and $^{94}_{38}\text{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are)

(a) $x = n, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(b) $x = p, y = e^-, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(c) $x = p, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(d) $x = n, y = n, K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

10. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string

being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(a) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$

(b) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$

(c) $\vec{V}_P \cdot \vec{V}_Q > 0$

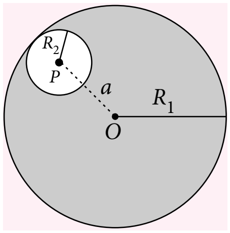
(d) $\vec{V}_P \cdot \vec{V}_Q < 0$

11. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$

(c) $I = \varepsilon_0 c V$ (d) $\mu_0 c I = \varepsilon_0 V$

12. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = $a = R_1 - R_2$ (see figure) is made.



If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)

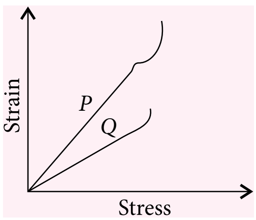
(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(d) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

13. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



(a) P has more tensile strength than Q

(b) P is more ductile than Q

(c) P is more brittle than Q

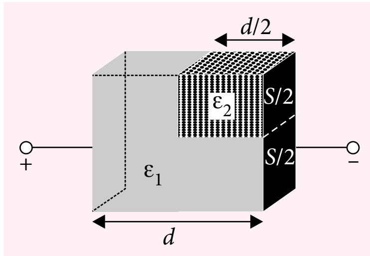
(d) The Young's modulus of P is more than that of Q

14. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is (are)

(a) $P(r = 0) = 0$ (b) $\dfrac{P(r = 3R/4)}{P(r = 2R/3)} = \dfrac{63}{80}$

(c) $\dfrac{P(r = 3R/5)}{P(r = 2R/5)} = \dfrac{16}{21}$ (d) $\dfrac{P(r = R/2)}{P(r = R/3)} = \dfrac{20}{27}$

15. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in figure, the capacitance becomes C_2. The ratio $\dfrac{C_2}{C_1}$



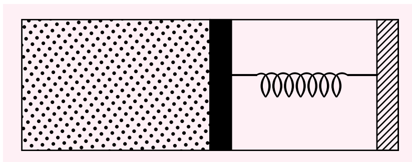
(a) 6/5

(b) 5/3

(c) 7/5

(d) 7/3

16. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\dfrac{1}{4}P_1V_1$

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\dfrac{7}{3}P_1V_1$

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\dfrac{17}{6}P_1V_1$

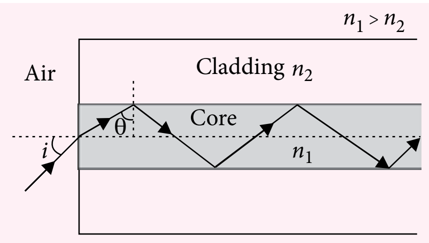
SECTION 3 (Maximum Marks : 16)

- *This section contains TWO paragraphs*
- *Based on each paragraph, there will be TWO questions*

- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four options(s) is(are) correct*
- *For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS*
- *Marking scheme :*
  - *+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened*
  - *0 If none of the bubbles is darkened*
  - *–2 In all other cases*

PARAGRAPH 1

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



17. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are)

(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

18. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

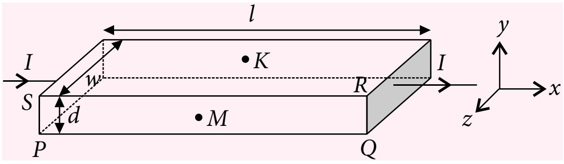
(a) $\frac{NA_1\, NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

PARAGRAPH 2

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

20. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5\,V_1$

(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

SOLUTIONS
PAPER-1

1. (7) : Case I : When system is kept in air.

Here, f_m = –10 cm, f_l = 10 cm

For reflection from concave mirror first,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_m};\ \frac{1}{v}+\frac{1}{-15}=\frac{1}{-10} \qquad [\because u=-15\text{ cm}]$$

$$\frac{1}{v}=\frac{1}{15}-\frac{1}{10}=-\frac{1}{30} \quad \therefore v = -30\text{ cm}$$

$$\text{Magnification, } m_1=-\frac{v}{u}=-\frac{-30}{-15}=-2$$

Now for refraction from lens, u = –(50 – 30) = –20 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_l} \Rightarrow \frac{1}{v}-\frac{1}{-20}=\frac{1}{10}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20} \quad \therefore v = 20\text{ cm}$$

$$\text{Magnification, } m_2=\frac{v}{u}=\frac{20}{-20}=-1$$

Magnification produced by the combination,

$$M_1 = m_1 \times m_2 = (-2) \times (-1) = 2$$

Case II : When system is kept in a medium of refractive index 7/6.

Image formed by the mirror does not get affected by surrounding medium.

$$\text{For lens, } \frac{1}{f_l'}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_l'}=\left(\frac{3/2}{7/6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_l'}=\frac{2}{7}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(i)$$

Also, when lens was in air

$$\frac{1}{f_l}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{10}$$

$$\therefore \left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{5}$$

Using this result in eqn. (i), we get

$$\frac{1}{f_l'}=\frac{2}{7}\times\frac{1}{5} \quad \therefore \quad f_l'=\frac{35}{2}\text{ cm}$$

$$\text{Again using lens formula, } \frac{1}{v}-\frac{1}{u}=\frac{1}{f_l'}$$

$$\frac{1}{v}-\frac{1}{-20}=\frac{2}{35} \Rightarrow \frac{1}{v}=\frac{2}{35}-\frac{1}{20}=\frac{1}{140}$$

$\therefore$ v = 140 cm

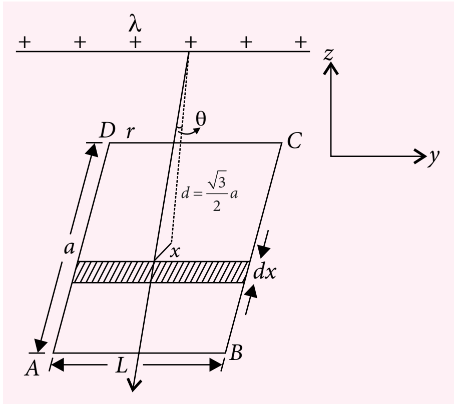
$$\text{Magnification, } m_2'=\frac{v}{u}=\frac{140}{-20}=-7$$

Magnification produced by the combination,

$M_2 = m_1 \times m_2' = (-2) \times (-7) = 14$

$$\therefore \left|\frac{M_2}{M_1}\right|=\frac{14}{2}=7$$

2. (6) : Consider a strip of thickness dx at a distance x from origin along x-axis.



Area of strip = $L\,dx$

Electric field on the strip due to line charge,

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}=\frac{\lambda}{2\pi\varepsilon_0\sqrt{d^2+x^2}}$$

Flux through strip,

$$d\phi=(E)\,(L\,dx)\cos\theta=\frac{\lambda\times(L\,dx)}{2\pi\varepsilon_0\sqrt{d^2+x^2}}\times\frac{d}{\sqrt{d^2+x^2}}$$

Net flux through the rectangular surface $ABCD$,

$$\phi=\int d\phi=\frac{\lambda L d}{2\pi\varepsilon_0}\int_{-a/2}^{a/2}\frac{dx}{d^2+x^2}$$

$$=\frac{\lambda L d}{2\pi\varepsilon_0}\left(\frac{1}{d}\right)\left[\tan^{-1}\frac{x}{d}\right]_{-a/2}^{a/2}$$

$$=\frac{\lambda L}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{a}{2d}\right)-\tan^{-1}\left(-\frac{a}{2d}\right)\right]$$

$$=\frac{\lambda L}{2\pi\varepsilon_0}\times 2\tan^{-1}\left(\frac{a}{2d}\right)$$

$$=\frac{\lambda L}{\pi\varepsilon_0}\tan^{-1}\left(\frac{a}{2\times\frac{\sqrt{3}}{2}a}\right)=\frac{\lambda L}{\pi\varepsilon_0}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\frac{\lambda L}{\pi\varepsilon_0}\left(\frac{\pi}{6}\right)=\frac{\lambda L}{6\varepsilon_0}=\frac{\lambda L}{n\varepsilon_0} \quad \therefore n=6$$

3. **(2) :** Wavelength of radiation, λ = 90 nm
Corresponding energy

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1242 \text{ eV nm}}{90 \text{ nm}} = 13.8 \text{ eV}$$

Kinetic energy of ejected electron, E_k = 10.4 eV
Energy required to ionize a hydrogen atom with its electron in n^{th} orbital,

$$E_{\text{ionize}} = \frac{13.6}{n^2} \text{ eV}$$

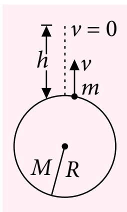
Using energy conservation principle,
$E_{\text{photon}} = E_{\text{ionize}} + E_k$

$$13.8 \text{ eV} = \frac{13.6}{n^2} \text{ eV} + 10.4 \text{ eV}$$

$$\frac{13.6}{n^2} = 3.4 \Rightarrow n^2 = \frac{13.6}{3.4} = 4 \therefore n = 2.$$

4. **(2) :** Given situation is shown in the figure.
Let acceleration due to gravity at the surface of the planet be g. At height h above planet's surface v = 0.
According to question,
acceleration due to gravity of the planet at height h above its surface becomes $g/4$.

$$g_h = \frac{g}{4} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$



$$4 = \left(1 + \frac{h}{R}\right)^2 \Rightarrow 1 + \frac{h}{R} = 2$$

$$\frac{h}{R} = 1 \Rightarrow h = R.$$

So, velocity of the bullet becomes zero at $h = R$.

$$\text{Also } v_{\text{esc}} = v\sqrt{N} \Rightarrow \sqrt{\frac{2GM}{R}} = v\sqrt{N} \quad \text{... (i)}$$

Applying energy conservation principle,
Energy of bullet at surface of earth
= Energy of bullet at highest point

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R} \therefore v = \sqrt{\frac{GM}{R}}$$

Putting this value in eqn. (i), we get

$$\sqrt{\frac{2GM}{R}} = \sqrt{\frac{NGM}{R}} \therefore N = 2$$

5. **(7) :** Suppose mass and radius of each disc are m and R respectively. Also potential energy at points B and D is zero *i.e.*, they are on reference line.
Given final kinetic energy for each disc is same, say it is K.
Applying energy conservation principle,
For surface AB,

$$\frac{1}{2}I_1\omega_1^2 + mg \times 30 = K \quad \text{...(i)}$$

For surface CD,

$$\frac{1}{2}I_2\omega_2^2 + mg \times 27 = K \quad \text{...(ii)}$$

From eqns. (i) and (ii), we get

$$\frac{1}{2}I_1\omega_1^2 + mg \times 30 = \frac{1}{2}I_2\omega_2^2 + mg \times 27 \quad \text{...(iii)}$$

$$\text{Here, } \omega_1 = \frac{v_1}{R}, \omega_2 = \frac{v_2}{R}, v_1 = 3 \text{ m s}^{-1}, v_2 = ?$$

$I_1 = I_2$ = Moment of inertia of disc about the point of contact

$$= \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

From eqn. (iii), $\frac{1}{2}\left(\frac{3}{2}mR^2\right) \times \left(\frac{3}{R}\right)^2 + m \times 10 \times 30$

$$= \frac{1}{2}\left(\frac{3}{2}mR^2\right)\left(\frac{v_2}{R}\right)^2 + m \times 10 \times 27$$

$$\frac{27}{4} + 300 = \frac{3}{4}v_2^2 + 270$$

$$\frac{3}{4}v_2^2 = \frac{27}{4} + 30 \Rightarrow 3v_2^2 = 147$$

$v_2^2 = 49 \therefore v_2 = 7 \text{ m s}^{-1}$

6. **(2) :** Here, $R_A = 400\, R_B$, $P_A = 10^4\, P_B$, $\frac{\lambda_A}{\lambda_B} = ?$

We know, $P = e\sigma AT^4$ ...(i)
λT = constant ...(ii)

$$\frac{P_A}{P_B} = \frac{e\sigma A_A T_A^{\,4}}{e\sigma A_B T_B^{\,4}} = \frac{A_A T_A^{\,4}}{A_B T_B^{\,4}}$$

$$\frac{P_A}{P_B} = \frac{(4\pi R_A^2)}{(4\pi R_B^2)} \times \left(\frac{\lambda_B}{\lambda_A}\right)^4 \quad \text{[Using eqn. (ii)]}$$

Plug in the given values

$$\frac{10^4 P_B}{P_B} = \frac{(400\, R_B)^2}{R_B^2} \times \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$\left(\frac{\lambda_A}{\lambda_B}\right)^4 = \frac{(400)^2}{10^4} = 16$$

$$\left(\frac{\lambda_A}{\lambda_B}\right)^4 = 2^4 \quad \therefore \frac{\lambda_A}{\lambda_B} = 2$$

7. **(3) :** Half life of radioactive material = T years

Let amount of radioactive material as fuel at the beginning be N_0 and corresponding power produced by it be P_0.

According to question,

Power requirement of the village

$$= 12.5\% \text{ of } P_0 = \frac{P_0}{8}$$

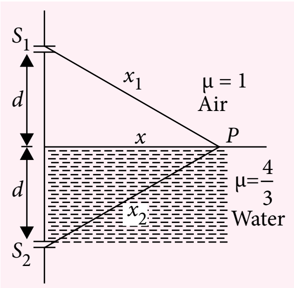
Since after each T year, power will be half, *i.e.*,

$$P_0 \xrightarrow{T} \frac{P_0}{2} \xrightarrow{T} \frac{P_0}{4} \xrightarrow{T} \frac{P_0}{8}$$

Total time upto which the plant can meet the village's need = $3T$ years = nT years

$\therefore \quad n = 3$

8. **(3) :** In the given figure,



$$S_1P = x_1 = 1 \times \sqrt{d^2 + x^2}$$

$$S_2P = x_2 = \frac{4}{3} \times \sqrt{d^2 + x^2}$$

Path difference between two waves at point P,

$\Delta x = x_2 - x_1$

$$= \left(\frac{4}{3} - 1\right)\sqrt{x^2 + d^2} = \frac{1}{3}\sqrt{x^2 + d^2}$$

Maxima will occur at point P if $\Delta x = m\lambda$

Here, m is an integer

$$\frac{1}{3}\sqrt{x^2 + d^2} = m\lambda \Rightarrow x^2 + d^2 = 9m^2\lambda^2$$

$\therefore \quad x^2 = 9m^2\lambda^2 - d^2 = p^2 m^2 \lambda^2 - d^2$

Hence, $p^2 = 9$ or $p = 3$.

9. **(a, c) :** Stopping potential (V_0) is given by

$$eV_0 = \frac{hc}{\lambda} - \phi$$

Graph between V_0 and λ :

$$eV_0 + \phi = \frac{hc}{\lambda}$$

$(eV_0 + \phi)\lambda = hc$

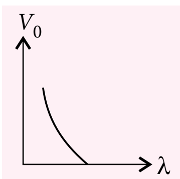
$(eV_0 + \phi)\lambda = \text{constant}$

Here, both e and ϕ are also constant.

It represents a hyperbola.

For $V_0 = 0$,

$$\lambda = \frac{\text{constant}}{\phi} = \text{constant}$$

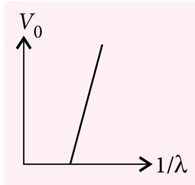


So correct option is (a).

Graph between V_0 and $\frac{1}{\lambda}$:

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \left(\frac{\phi}{e}\right)$$

It represents a straight line with slope $\left(\frac{hc}{e}\right)$ and intercept $\left(-\frac{\phi}{e}\right)$ on V_0 axis.



So correct option is (c).

10. **(b, c) :** For vernier callipers,

$$8 \text{ M.S.D.} = 1 \text{ cm}; \quad 1 \text{ M.S.D.} = \frac{1}{8} \text{ cm}$$

5 V.S.D. = 4 M.S.D.

$$1 \text{ V.S.D.} = \frac{4}{5} \text{ M.S.D.} = \frac{4}{5} \times \frac{1}{8} \text{ cm} = \frac{1}{10} \text{ cm}$$

LC of vernier callipers = 1 M.S.D. – 1 V.S.D.

$$= \frac{1}{8} - \frac{1}{10} = \frac{1}{40} \text{ cm}$$

For screw gauge, $\text{LC} = \frac{\text{Pitch}}{100} = \frac{P}{100}$

One complete rotation of circular scale = two divisions on the linear scale

If $P = 2 \times$ LC of vernier callipers, then

$$P = 2 \times \frac{1}{40} \text{ cm} = \frac{1}{20} \text{ cm}$$

$$\therefore \quad \text{LC of screw gauge} = \frac{P}{100} = \frac{1}{2000} \text{ cm} = \frac{1}{200} \text{ mm}$$

$= 0.005$ mm

So, option (b) is correct.

If LC of linear scale of screw gauge

= 2 × LC of vernier callipers

$$= 2 \times \frac{1}{40} = \frac{1}{20} \text{ cm}$$

$$\text{So pitch } = 2 \times \frac{1}{20} = \frac{1}{10} \text{ cm} = 1 \text{ mm}$$

$$\text{LC of screw gauge} = \frac{1 \text{ mm}}{100} = 0.01 \text{ mm}$$

So, option (c) is correct.

11. **(a, c, d) :** Here, Planck's constant h, speed of light c and gravitational constant G are used as basic units for length L and Mass M.

So, $L \propto h^x c^y G^z$ ...(i)

and $M \propto h^p c^q G^r$ ...(ii)

Also, $[h] = [\text{M L}^2 \text{T}^{-1}]$, $[c] = [\text{L T}^{-1}]$

$[G] = [\text{M}^{-1} \text{L}^3 \text{T}^{-2}]$

Using principle of homogeneity of dimensions

For eqn. (i)

$[M^0 L T^0] = [M^x L^{2x} T^{-x}][L^y T^{-y}][M^{-z} L^{3z} T^{-2z}]$
$M^0 L T^0 = M^{(x-z)} L^{(2x+y+3z)} T^{(-x-y-2z)}$
On comparing powers from both sides, we get
$x - z = 0,\ 2x + y + 3z = 1,\ -x - y - 2z = 0$
On solving these eqns., we get

$$x = \frac{1}{2},\ y = -\frac{3}{2},\ z = \frac{1}{2}$$

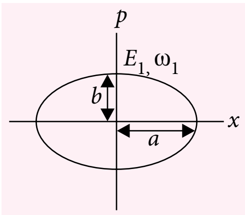
$$\therefore \quad L = K\sqrt{\frac{hG}{c^3}};\ K \text{ is some constant.}$$

For eqn. (ii)
$[M L^0 T^0] = [M^p L^{2p} T^{-p}][L^q T^{-q}][M^{-r} L^{3r} T^{-2r}]$
$M L^0 T^0 = M^{(p-r)} L^{(2p+q+3r)} T^{(-p-q-2r)}$
On comparing powers from both sides, we get
$p - r = 1,\ 2p + q + 3r = 0,\ -p - q - 2r = 0$
On solving these eqns., we get

$$p = \frac{1}{2},\ q = \frac{1}{2},\ r = -\frac{1}{2}$$

$$\therefore \quad M = K'\sqrt{\frac{hc}{G}};\ K' \text{ is some constant.}$$

12. (b, d) : For first oscillator,
Angular frequency = ω_1
Amplitude = a
Total energy = E_1
Maximum momentum,
$p_{max} = b$

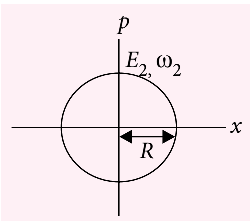


$$E_1 = \frac{1}{2} m \omega_1^2 a^2 \qquad \text{...(i)}$$

$p_{max} = mv_{max} = m\, a\, \omega_1 \Rightarrow b = m\, a\, \omega_1$

$$\frac{a}{b} = \frac{1}{m\omega_1} \qquad \text{...(ii)}$$

For second oscillator,
Angular frequency = ω_2
Amplitude = R
Maximum momentum,
$p_{max} = R$
Total energy = E_2



$$E_2 = \frac{1}{2} m\omega_2^2 R^2 \qquad \text{...(iii)}$$

$p_{max} = mv_{max} = m\omega_2 R$
$R = m\omega_2 R \Rightarrow m\omega_2 = 1$ ...(iv)
From eqns. (ii) and (iv),

$$\frac{a}{b} = \frac{\omega_2}{\omega_1} \qquad \text{...(v)}$$

From (i) and (iii),

$$\frac{E_1}{E_2} = \frac{\omega_1^2 a^2}{\omega_2^2 R^2} \qquad \text{...(vi)}$$

If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then from eqn. (v)

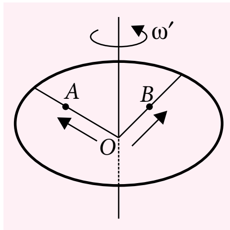
$$\frac{\omega_2}{\omega_1} = n^2$$

and from eqn. (vi)

$$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} \times n^2 = \frac{\omega_1}{\omega_2} \quad \therefore \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

So, options (b) and (d) are correct.

13. (d) : Ring has mass M and radius R.
Initial angular speed of ring = ω
Two point masses, each of mass $\frac{M}{8}$ are at rest at O.
Initial angular momentum of ring and point masses system,
$L_i = I_R\omega + I_m\omega + I_m\omega$
$= MR^2\omega + 0 + 0 = MR^2\omega$
After some time, situation is changed as shown in the figure.



Angular speed of the system,

$$\omega' = \frac{8}{9}\omega;\ OA = \frac{3R}{5};$$

$OB = r = ?$
Moment of inertia about O of point mass at A,

$$I_A = \frac{M}{8} \times \frac{9R^2}{25}$$

Moment of inertia about O of point mass at B,

$$I_B = \frac{M}{8} r^2$$

Final angular momentum of the system
$L_f = MR^2\omega' + I_A\omega' + I_B\omega'$

$$= MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8} r^2 \times \frac{8\omega}{9}$$

As there is no external torque acting on the system so its angular momentum will be conserved, $L_i = L_f$

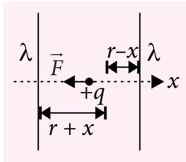
$$MR^2\omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8} r^2 \times \frac{8\omega}{9}$$

$$R^2 = \frac{8R^2}{9} + \frac{R^2}{25} + \frac{r^2}{9} \Rightarrow \frac{r^2}{9} = \frac{16}{225} R^2 \quad \therefore r = \frac{4}{5} R$$

14. (c) : Case I : Charge q is displaced towards right by a very small distance x.
Net force acting on the charge particle,

$$\vec{F} = \frac{\lambda q}{2\pi\varepsilon_0 (r + x)} \hat{i} + \frac{\lambda q}{2\pi\varepsilon_0 (r - x)} (-\hat{i})$$

$$= \frac{\lambda q}{2\pi\varepsilon_0}\left[\frac{r-x-r-x}{r^2-x^2}\right](\hat{i})$$



$$\vec{F} = -\frac{\lambda q}{\pi\varepsilon_0 r^2}x(\hat{i})\ [\because x << r]$$

or $F = -\omega^2 x$, which is S.H.M. equation.

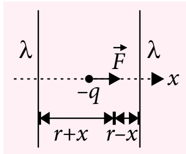
Hence charge $+q$ will perform S.H.M. when displaced by a small distance.

Case II : Charge $-q$ is displaced towards right by a very small distance x.

Net force acting on the charged particle

$$\vec{F} = \frac{-\lambda q}{2\pi\varepsilon_0 (r+x)}(\hat{i}) + \frac{-\lambda q}{2\pi\varepsilon_0 (r-x)}(-\hat{i})$$

$$= \frac{-\lambda q}{2\pi\varepsilon_0}\left[\frac{r-x-r-x}{r^2-x^2}\right](\hat{i})$$



$$\vec{F} = \frac{\lambda q}{\pi\varepsilon_0 r^2}x(\hat{i})\quad [\because r >> x]$$

$F \propto x$

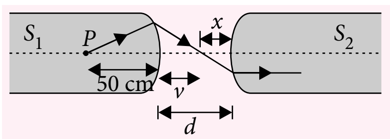
Hence, charge $-q$ continues to move in the direction of its displacement.

15. (b) : For glass rod S_1,

$R = -10$ cm, $\mu_1 = 1.5$, $\mu_2 = 1$, $u = -50$ cm, $v_1 = ?$

I[st] refraction at curved surface of S_1 from denser medium to rarer medium,

using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$$\frac{1}{v_1} - \frac{1.5}{-50} = \frac{1-1.5}{-10} \Rightarrow \frac{1}{v_1} = \frac{1}{20} - \frac{3}{100} = \frac{1}{50}$$

$\therefore\ v_1 = 50$ cm

For glass rod S_2,

$R = 10$ cm, $u = -x$, $v = \infty$, $\mu_1 = 1$, $\mu_2 = 1.5$

II[nd] refraction at curved surface of S_2 from rarer medium to denser,

using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10} = \frac{1}{20}$$

$$\frac{1}{x} = \frac{1}{20};\quad \therefore\ x = 20\text{ cm}$$

Separation between glass rods is given by

$d = v_1 + x = 50 + 20 = 70$ cm

16. (a, b, c) : Magnetic force acting on a current carrying wire, kept in a uniform magnetic field,

$\vec{F} = I(\vec{l} \times \vec{B})$

Here, $\vec{l}$ = displacement of the wire $= 2(L+R)\hat{x}$

$\therefore\quad \vec{F} = 2I(L+R)(\hat{x} \times \vec{B})$

If $\vec{B} = B\hat{x}$ then

$\vec{F} = 2I(L+R)(\hat{x}\times\hat{x})B = 0$

If $\vec{B} = B\hat{y}$ then

$\vec{F} = 2I(L+R)(\hat{x}\times\hat{y})B = 2IB(L+R)\hat{z}$

or $F \propto (L+R)$

If $\vec{B} = B\hat{z}$ then

$\vec{F} = 2I(L+R)(\hat{x}\times\hat{z})B = -2IB(L+R)\hat{y}$

or, $F \propto (L+R)$.

So, options (a), (b) and (c) are correct.

17. (a, b, d) : For hydrogen, $n_1 = 1, C_{V_1} = \frac{5}{2}R$

For helium, $n_2 = 1, C_{V_2} = \frac{3}{2}R$

For mixture of gases, $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$$= \frac{1\times\frac{5}{2}R + 1\times\frac{3}{2}R}{1+1} = 2R$$

$C_P = C_V + R = 3R,\ \gamma_{\text{mix}} = \frac{C_P}{C_V} = \frac{3}{2}$

Also, $\gamma_{\text{mix}} = 1 + \frac{2}{f} \Rightarrow \frac{3}{2} = 1 + \frac{2}{f}\ \therefore\ f = 4$

$\therefore$ Average energy per mole $= \frac{1}{2}fRT = 2RT$

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{1\times 2 + 1\times 4}{1+1} = 3\text{ g/mol}$$

Speed of sound in a gas, $v = \sqrt{\frac{\gamma RT}{M}}$

For a given value of T, $v \propto \sqrt{\frac{\gamma}{M}}$

$$\frac{v_{\text{mix}}}{v_{\text{He}}} = \sqrt{\frac{\gamma_{\text{mix}}}{\gamma_{\text{He}}}\times\frac{M_{\text{He}}}{M_{\text{mix}}}} = \sqrt{\frac{3}{2}\times\frac{3}{5}\times\frac{4}{3}} = \sqrt{\frac{6}{5}}$$

rms speed of a gas molecule at temperature T is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

For a given value of T, $v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$

$$\frac{(v_{rms})_{He}}{(v_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{He}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

So, options (a), (b) and (d) are correct.

18. (b) : Resistance of a wire, $R = \frac{\rho l}{A}$

For iron (Fe) bar,

$\rho = 10^{-7}\ \Omega$ m, l = 50 mm = 50×10^{-3} m

A = (2 mm) × (2 mm) = 4 mm^2 = 4×10^{-6} m^2

$$R_1 = \frac{10^{-7} \times 50 \times 10^{-3}}{4 \times 10^{-6}} = 1250 \times 10^{-6}\ \Omega = 1250\ \mu\Omega$$

For aluminium (Al) bar,

$\rho = 2.7 \times 10^{-8}\ \Omega$ m, l = 50 mm = 50×10^{-3} m

$A = (7^2 - 2^2)$ mm^2 = 45 mm^2 = 45×10^{-6} m^2

$$\therefore \quad R_2 = \frac{2.7 \times 10^{-8} \times 50 \times 10^{-3}}{45 \times 10^{-6}}$$

$$= \frac{27 \times 50}{45} \times 10^{-6} = 30 \times 10^{-6}\ \Omega = 30\ \mu\Omega$$

Potential difference across both bars (resistors) is same so they are in parallel combination.

Equivalent resistance between P and Q is given by

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{1250 \times 30}{1250 + 30} = \frac{125 \times 30}{128} = \frac{1875}{64}\ \mu\Omega.$$

***19.** (A) → (R) or (R, T); (B) → (P, S); (C) → (Q, T); (D) → (R)

[*As per official answer key published on jeeadv.iitb.ac.in*]

20. $U_1(x) = \frac{U_0}{2}\left[1 - \frac{x^2}{a^2}\right]^2$

$$F_1 = -\frac{dU_1}{dx} = -\frac{U_0}{2} \times 2\left[1 - \frac{x^2}{a^2}\right]\left[-\frac{2x}{a^2}\right]$$

$$= \frac{2U_0}{a^4}(a^2 - x^2)x = \frac{2U_0}{a^4}(a + x)(a - x)x$$

$F_1 = 0$ at $x = 0, a, -a$

At $x = \pm a$, $U_1 = 0$, $x = 0$, $U_1 = \frac{U_0}{2}$

Hence, particle will oscillate if total energy is less than $U_0/2$.

(A) → (P, Q, R, T)

$$U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$$

$$F_2 = -\frac{dU_2}{dx} = -\frac{U_0}{2} \times 2 \times \left(\frac{x}{a}\right) \times \frac{1}{a} = -\frac{U_0 x}{a^2}$$

So, $F_2 = 0$ at $x = 0$

At $x = a$, $F_2 = -\frac{U_0}{a}$

At $x = -a$, $F_2 = \frac{U_0}{a}$

(B) → (Q, S)

$$U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$$

$$F_3 = -\frac{dU_3}{dx}$$

$$= -\frac{U_0}{2}\left[\frac{2x}{a^2}\exp\left(-\frac{x^2}{a^2}\right) + \frac{x^2}{a^2}\exp\left(-\frac{x^2}{a^2}\right)\left(-\frac{2x}{a^2}\right)\right]$$

$$= -\frac{U_0}{2}\frac{2x}{a^4}\exp\left(-\frac{x^2}{a^2}\right)[a^2 - x^2]$$

$$= -\frac{U_0}{a^4}(a^2 - x^2)x\exp\left(-\frac{x^2}{a^2}\right)$$

$$F_3 = \frac{U_0}{a^4}[x(x - a)(x + a)]\exp\left(-\frac{x^2}{a^2}\right)$$

$F_3 = 0$ at $x = 0, a, -a$

At $x = 0$, $U_3 = 0$

(C) → (P, Q, R, S)

$$U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$$

$$F_4 = -\frac{dU_4}{dx} = -\frac{U_0}{2}\left[\frac{1}{a} - \frac{x^2}{a^3}\right] = \frac{U_0}{2a^3}(x^2 - a^2)$$

$F_4 = 0$ at $x = a, -a$

At $x = -a$, $U_4 = -\frac{U_0}{3}$; $x = a$, $U_4 = \frac{U_0}{3}$; $x = 0$, $U_4 = 0$

Hence, particle will oscillate about $x = -a$ if total energy is less than $U_0/3$.

(D) → (P, R, T)

PAPER-2

1. (2) : Here, angular momentum of an electron in an excited state of $Li^{2+} = \frac{3h}{2\pi}$

de Broglie wavelength of electron in this state = $p\pi a_0$

$Z = 3$, p = ?

From Bohr's II postulate, $mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi}$ $\therefore n = 3$

and momentum, $mv = \frac{3h}{2\pi r}$ ...(i)

Now, radius of n^{th} shell, $r = \left(\frac{n^2}{Z}\right)a_0 = \left(\frac{3^2}{3}\right)a_0$

$r = 3a_0$ ...(ii)

de Broglie wavelength of the electron

$$\lambda = \frac{h}{mv} = \frac{h}{\frac{3h}{2\pi r}} = \frac{2\pi r}{3} \quad \text{[Using eqn. (i)]}$$

$$= \frac{2\pi}{3} \times 3a_0 = 2\pi a_0 \quad \text{[Using eqn. (ii)]}$$

$\therefore$ $\lambda = 2\pi a_0 = p\pi a_0$ Hence, $p = 2$.

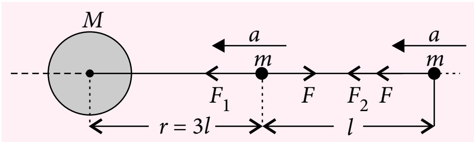
2. **(7) :** Both the point masses are connected by a light rod so they have same acceleration.

Suppose each point mass is moving with acceleration a towards larger mass M.

Using Newton's 2nd law of motion for point mass nearer to larger mass,

$F_1 - F = ma$

$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma \quad \text{...(i)}$$



Again using 2nd law of motion for another mass

$F_2 + F = ma$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma \quad \text{...(ii)}$$

From eqn. (i) and (ii), we get

$$\frac{GM}{9l^2} - \frac{Gm}{l^2} = \frac{GM}{16l^2} + \frac{Gm}{l^2}$$

$$\frac{M}{9} - \frac{M}{16} = m + m \Rightarrow \frac{7M}{144} = 2m$$

$$m = \frac{7M}{288} = k\left(\frac{M}{288}\right) \quad \therefore k = 7$$

3. **(4) :** $E(t) = A^2 e^{-\alpha t}$

Taking natural logarithm on both sides,

$\ln(E) = 2\ln(A) + (-\alpha t)$

Differentiating both sides

$$\frac{dE}{E} = 2\left(\frac{dA}{A}\right) + (-\alpha dt)$$

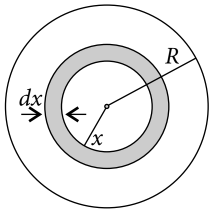
Errors always add up for maximum error.

$$\therefore \frac{dE}{E} = 2\frac{dA}{A} + \alpha\left(\frac{dt}{t}\right) \times t$$

Here, $\frac{dA}{A} = 1.25\%, \frac{dt}{t} = 1.5\%, t = 5\text{ s}, \alpha = 0.2\text{ s}^{-1}$

$$\therefore \frac{dE}{E} = (2 \times 1.25\,\%) + (0.2) \times (1.5\%) \times 5 = 4\,\%$$

4. **(6) :** For solid sphere A,



$$\rho_A(r) = k\left(\frac{r}{R}\right)$$

Consider a spherical shell of radius x and thickness dx.

Mass of the shell, dm = density × volume

$$= \left(k\frac{x}{R}\right)(4\pi x^2\,dx)$$

So, moment of inertia of shell about its diameter,

$$dI = \frac{2}{3}(dm)x^2 = \frac{2}{3}\left(k\frac{x}{R}\right)(4\pi x^2\,dx)x^2$$

$$= \left(\frac{8\pi}{3}\frac{k}{R}\right)x^5\,dx$$

$\therefore$ Moment of inertia of the sphere A, $I_A = \int_0^R dI$

$$= \frac{8\pi k}{3R}\int_0^R x^5 dx = \frac{8\pi k}{3R}\left[\frac{x^6}{6}\right]_0^R$$

$$I_A = \left(\frac{8\pi k}{18}\right)R^5 \quad \text{...(i)}$$

Similarly, for sphere B

$$I_B = \frac{8\pi k}{3R^5}\int_0^R x^9 dx = \left(\frac{8\pi k}{3R^5}\right)\left[\frac{x^{10}}{10}\right]_0^R$$

$$\therefore \quad I_B = \frac{8\pi k}{30}R^5 \quad \text{...(ii)}$$

From eqns. (i) and (ii), we get

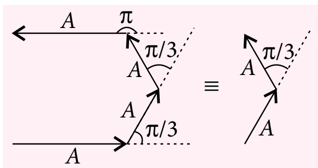
$$\frac{I_B}{I_A} = \frac{18}{30} = \frac{6}{10} = \frac{n}{10} \quad \therefore n = 6$$

5. **(3) :** Intensity of each wave = I_0.

Let amplitude of each wave be A.

$A = \sqrt{I_0}$ ...(i)

Using phasor method to show superposition of waves.



Clearly, first and fourth waves interfere destructively.

So, resultant amplitude of 2nd and 3rd waves is given by

$$A_r = \sqrt{A^2 + A^2 + 2A^2\cos 60°} = \sqrt{3A^2}$$

$A_r^2 = 3A^2$

$I_r = 3I_0 = nI_0 \quad \therefore\ n = 3$ [Using eqn. (i)]

6. **(2) :** Number of nuclei present at time t,

$N(t) = N_0 e^{-\lambda t}$

$$\tau_P = \tau = \frac{1}{\lambda_P} \text{ and } \tau_Q = 2\tau = \frac{1}{\lambda_Q} \quad ...(i)$$

According to question,

$$A = -\frac{dN}{dt} = N_0\, \lambda\, e^{-\lambda t};\ R = -\frac{dA}{dt} = N_0 \lambda^2\, e^{-\lambda t}$$

Also, at $t = 0$, $A_P = A_Q$

$\lambda_P N_P\, e^{-\lambda_P \times 0} = \lambda_Q\, N_Q\, e^{-\lambda_Q \times 0}$

$\lambda_P\, N_P = \lambda_Q\, N_Q$ ...(ii)

At time $t = 2\tau$

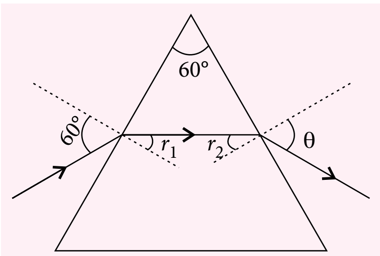
$$\frac{R_P}{R_Q} = \frac{N_P}{N_Q}\left(\frac{\lambda_P}{\lambda_Q}\right)^2 \times \frac{e^{-\lambda_P 2\tau}}{e^{-\lambda_Q 2\tau}}$$

$$= \left(\frac{N_P\,\lambda_P}{N_Q\,\lambda_Q}\right)\left(\frac{\lambda_P}{\lambda_Q}\right)\frac{e^{-2(\lambda_P \tau)}}{e^{-2(\lambda_Q \tau)}} \quad ...(iii)$$

From eqns. (i), (ii) and (iii), we get

$$\frac{R_P}{R_Q} = 1 \times 2 \times \frac{e^{-2\times 1}}{e^{-2\times(1/2)}} = \frac{2}{e} = \frac{n}{e} \quad \therefore\ n = 2$$

7. **(2) :**



Using Snell's law at the first interface,

$$1 \sin 60° = n \sin r_1 \ \Rightarrow\ \sin r_1 = \frac{\sqrt{3}}{2n} \quad ...(i)$$

$$\cos r_1 = \sqrt{1 - \sin^2 r_1} = \sqrt{1 - \frac{3}{4n^2}} = \frac{\sqrt{4n^2 - 3}}{2n} \quad ...(ii)$$

Also, $r_1 + r_2 = A = 60°$ ...(iii)

Using Snell's law at the emerging interface,

$n \sin r_2 = 1 \sin \theta$

$n \sin (60° - r_1) = \sin \theta$ [Using eqn. (iii)]

$$n\left[\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1\right] = \sin\theta$$

$$n\left[\frac{\sqrt{3}}{2}\,\frac{\sqrt{4n^2 - 3}}{2n} - \frac{1}{2}\,\frac{\sqrt{3}}{2n}\right] = \sin\theta$$

[Using eqns.(i) and (ii)]

$$\frac{\sqrt{3}}{4}\left(\sqrt{4n^2 - 3} - 1\right) = \sin\theta$$

Differentiating both sides with respect to n

$$\frac{\sqrt{3}}{4}\left(\frac{1}{\sqrt{4n^2 - 3}} \times \frac{8n}{2} - 0\right) = \cos\theta\,\frac{d\theta}{dn}$$

$$\frac{d\theta}{dn} = \frac{\sqrt{3}\,n}{\sqrt{4n^2 - 3}} \times \frac{1}{\cos\theta}$$

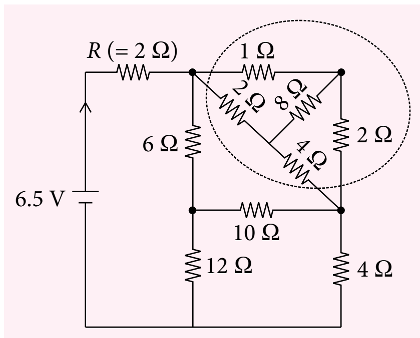
For $n = \sqrt{3}$, $\theta = 60°$

$$\frac{d\theta}{dn} = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{4\times 3 - 3}} \times \frac{1}{\cos 60°} = 2 = m$$

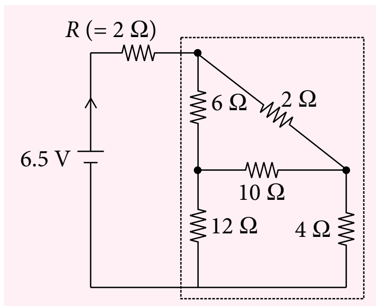
$\therefore \quad m = 2$

8. **(1) :** A portion of circuit which is enclosed, as shown in the figure forms balanced Wheatstone bridge. So no current will flow through 8 Ω and equivalent resistance of the bridge is given by

$$R_1 = \frac{(1+2)\times(2+4)}{(1+2)+(2+4)} = 2\ \Omega$$



This circuit reduces to equivalent circuit as shown in the figure.

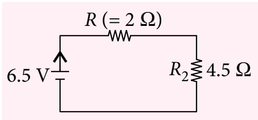


Again a portion of circuit, enclosed by dotted lines, forms a Wheatstone bridge. So no current will flow through 10 Ω and equivalent resistance of the bridge is given by

$$R_2 = \frac{(6+12)\times(2+4)}{(6+12)+(2+4)} = \frac{9}{2}\ \Omega = 4.5\ \Omega$$

Finally circuit reduces to a simple circuit as shown in the figure, Current through R,

$$I = \frac{6.5}{2+4.5} = 1\text{ A}$$



9. (a) : $^{236}_{92}\text{U} \longrightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$

$K_x = 2$ MeV, $K_y = 2$ MeV, $K_{Xe} = ?$, $K_{Sr} = ?$

By conservation of charge number and mass number, $x \equiv y \equiv n$

B.E. per nucleon of $^{236}_{92}\text{U} = 7.5$ MeV

B.E. per nucleon of $^{140}_{54}\text{Xe}$ or $^{94}_{38}\text{Sr} = 8.5$ MeV

Q value of reaction,

Q = Net kinetic energy gained in the process

$= K_{Xe} + K_{Sr} + 2 + 2 - 0 = K_{Xe} + K_{Sr} + 4$ ...(i)

As number of nucleons is conserved in a reaction, so Q = Difference of binding energies of the nuclei

$= 140 \times 8.5 + 94 \times 8.5 - 236 \times 7.5 = 219$ MeV ...(ii)

From eqns. (i) and (ii)

$K_{Xe} + K_{Sr} = 219 - 4 = 215$ MeV

Xe and Sr have momentum of same magnitude but in opposite directions.

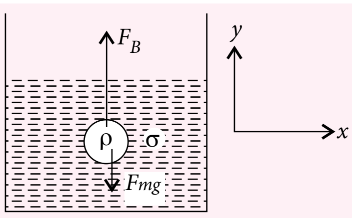
Hence, lighter body has larger kinetic energy.

So, from options,

$K_{Sr} = 129$ MeV, and $K_{Xe} = 86$ MeV

Hence, option (a) is correct.

10. (a, d) : Consider a body of density ρ kept in a fluid of density σ. Co-efficient of viscosity and terminal velocity of fluid are η and V respectively. As body is moving with terminal velocity, so net force on it will be zero.

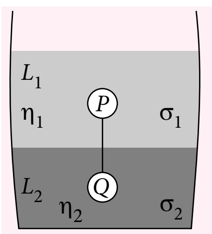


$$\vec{F}_{\text{viscous}} + \vec{F}_{mg} + \vec{F}_B = 0$$

$$\vec{F}_{\text{viscous}} + \rho\frac{4}{3}\pi R^3(-\hat{j}) + \sigma\frac{4}{3}\pi R^3(\hat{j}) = 0$$

$$\vec{F}_{\text{viscous}} = -6\pi\eta R\vec{V} = \frac{4}{3}\pi R^3(\rho - \sigma)(\hat{j}) \quad \therefore V \propto \frac{1}{\eta}$$

If $\rho > \sigma$ then $\vec{F}_{\text{viscous}}$ and $\vec{V}$ will be in upward and downward direction respectively and if $\rho < \sigma$ then $\vec{F}_{\text{viscous}}$ and $\vec{V}$ will be in downward and upward direction respectively.



As per given diagram,

$\sigma_2 > \sigma_1$, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$

$\Rightarrow \rho_2 > \sigma_2 > \sigma_1 > \rho_1$

Now, if we put sphere P in liquid L_2 where $|\vec{V}_P| \propto \dfrac{1}{\eta_2}$ and $\rho_1 < \sigma_2$ then $\vec{F}_{\text{viscous}}$ will be downward and $\vec{V}_P$ will be upward.

If we put sphere Q in liquid L_1 where $|\vec{V}_Q| \propto \dfrac{1}{\eta_1}$ and $\rho_2 > \sigma_1$ then $\vec{F}_{\text{viscous}}$ will be upward and $\vec{V}_Q$ will be downward.

Hence we can conclude that,

$$\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2} \text{ and } \vec{V}_P \cdot \vec{V}_Q < 0.$$

11. (a, c) : Using, $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $R = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$

check the dimensional correctness of equations.

(a) $\mu_0 I^2 = \varepsilon_0 V^2$

$$\frac{\mu_0}{\varepsilon_0} = \frac{V^2}{I^2} = R^2$$

$\therefore$ $R^2 = R^2$ which is dimensionally correct.

(b) $\varepsilon_0 I = \mu_0 V \Rightarrow \dfrac{\varepsilon_0}{\mu_0} = \dfrac{V}{I} \Rightarrow \dfrac{1}{R^2} = R$

which is dimensionally incorrect.

(c) $I = \varepsilon_0 cV \Rightarrow \dfrac{I}{V} = \varepsilon_0 c = \dfrac{\varepsilon_0}{\sqrt{\varepsilon_0\mu_0}}$

$$\frac{1}{R} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \Rightarrow \frac{1}{R} = \frac{1}{R}$$

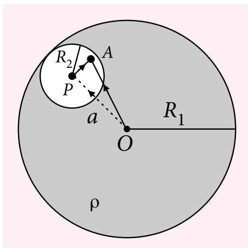
which is dimensionally correct.

(d) $\mu_0 cI = \varepsilon_0 V \Rightarrow \dfrac{\mu_0 c}{\varepsilon_0} = \dfrac{V}{I}$

$$\frac{\mu_0}{\varepsilon_0}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = R \Rightarrow \frac{1}{\varepsilon_0}\sqrt{\frac{\mu_0}{\varepsilon_0}} = R$$

$\dfrac{R}{\varepsilon_0} = R$, which is dimensionally incorrect.

12. (d) : Consider a point A inside the cavity where electric field has to be determined.



Charge is uniformly distributed in the sphere, let ρ be charge density.
Using superposition principle,
Electric field at A = Electric field at A due to sphere without cavity – Electric field at A due to the sphere of radius R_2 and centred at P

$$\therefore \quad \vec{E}_A = \frac{\rho}{3\varepsilon_0}\overrightarrow{OA} - \frac{\rho}{3\varepsilon_0}(\overrightarrow{PA}) = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OA} - \overrightarrow{PA})$$

$$\because \quad \overrightarrow{OP} + \overrightarrow{PA} = \overrightarrow{OA}, \ \overrightarrow{OA} - \overrightarrow{PA} = \overrightarrow{OP}$$

$$\therefore \quad \vec{E}_A = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OP}) = \frac{\rho}{3\varepsilon_0}(\vec{a})$$

Hence electric field is uniform inside the cavity and both its magnitude and direction depend on $\vec{a}$.

13. (a, b) : We know, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

According to graph,

$$\text{Slope of curve} = \frac{\text{Change in strain}}{\text{Change in stress}} = \frac{1}{Y}$$

$(\text{Slope})_P > (\text{Slope})_Q \therefore Y_P < Y_Q$

P has more tensile strength than Q as it sustains more stress after elastic limit.

There is large deformation between the elastic limit and the fracture point for material P as compared to material Q. Hence, P is more ductile than Q.

After the elastic limit, Q breaks soon as compared to P. So, Q is more brittle than P.

14. (b, c) : Let the density of fluid be ρ.

Pressure at a distance r ($r < R$) from centre is given by

$$P(r) = \frac{2\pi}{3}\rho^2 G(R^2 - r^2)$$

where G is gravitational constant.

Now, $P(r = 0) = \dfrac{2\pi}{3}\rho^2 GR^2 \neq 0$

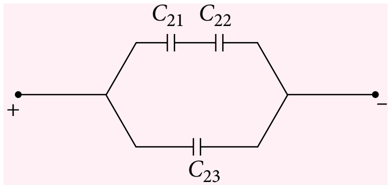
$$\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{R^2 - (3R/4)^2}{R^2 - (2R/3)^2} = \frac{7}{16} \times \frac{9}{5} = \frac{63}{80}$$

$$\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{R^2 - (3R/5)^2}{R^2 - (2R/5)^2} = \frac{16}{25} \times \frac{25}{21} = \frac{16}{21}$$

$$\frac{P(r = R/2)}{P(r = R/3)} = \frac{R^2 - (R/2)^2}{R^2 - (R/3)^2} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32} \neq \frac{20}{27}$$

15. (d) : $C_1 = \dfrac{\varepsilon_0 S}{d}$

After insertion of two dielectrics between plates, it becomes a combination of three capacitors as shown in the figure.



$$C_{21} \rightarrow \varepsilon_1 = 2, \frac{S}{2}, \frac{d}{2}; C_{21} = \frac{2\varepsilon_0 \frac{S}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 S}{d} = 2C_1$$

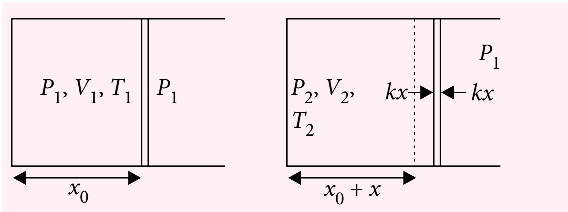
$$C_{23} \rightarrow \varepsilon_1 = 2, \frac{S}{2}, d; C_{23} = \frac{2\varepsilon_0 \frac{S}{2}}{d} = \frac{\varepsilon_0 S}{d} = C_1$$

$$C_{22} \rightarrow \varepsilon_2 = 4, \frac{S}{2}, \frac{d}{2}; C_{22} = \frac{4\varepsilon_0 \frac{S}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 S}{d} = 4C_1$$

$$\therefore \quad C_2 = C_{23} + \frac{C_{21} \times C_{22}}{C_{21} + C_{22}} = C_1 + \frac{2C_1 \times 4C_1}{2C_1 + 4C_1}$$

$$C_2 = C_1 + \frac{4}{3}C_1 = \frac{7}{3}C_1 \ \therefore \ \frac{C_2}{C_1} = \frac{7}{3}.$$

16. (a, b, c) : Initially both the compartments has same pressure as they are in equilibrium.



Suppose spring is compressed by x on heating the gas.

Let A be the area of cross-section of piston.

As gas is ideal monoatomic, so

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{...(i)}$$

Force on spring by gas = kx

$$\therefore \quad P_2 = P_1 + \frac{kx}{A} \quad \text{...(ii)}$$

Case I : When $V_2 = 2V_1$, $T_2 = 3T_1$

From eqn. (i)

$$\frac{P_1V_1}{T_1} = \frac{P_2(2V_1)}{3T_1} \ \Rightarrow \ P_2 = \frac{3}{2}P_1$$

Putting this value in eqn. (ii) we get

$$\frac{3}{2}P_1 = P_1 + \frac{kx}{A} \ \Rightarrow \ kx = \frac{P_1A}{2}$$

$$x = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$$

Energy stored in the spring

$$=\frac{1}{2}kx^2=\frac{1}{2}(kx)(x)=\frac{P_1V_1}{4}$$

So, option (a) is correct.

Change in internal energy,

$$\Delta U=\frac{f}{2}(P_2V_2-P_1\,V_1)=\frac{3}{2}\left(\frac{3}{2}P_1\times 2V_1-P_1V_1\right)$$

$$=3\,P_1V_1$$

So, option (b) is correct.

Case II : When $V_2=3V_1$ and $T_2=4T_1$

From eqn. (i),

$$\frac{P_1V_1}{T_1}=\frac{P_2(3V_1)}{4T_1}\;\Rightarrow\;P_2=\frac{4}{3}P_1$$

$$x=\frac{V_2-V_1}{A}=\frac{2V_1}{A}$$

From eqn. (ii),

$$\frac{4}{3}P_1=P_1+\frac{kx}{A}\;\Rightarrow\;kx=\frac{P_1A}{3}$$

Gas is heated very slowly so pressure on the other compartment remains same.

Work done by gas = Work done by gas on atmosphere + Energy stored in spring.

$$W_g=P_1Ax+\frac{1}{2}kx^2=P_1(2V_1)+\frac{1}{2}\left(\frac{P_1A}{3}\right)\left(\frac{2V_1}{A}\right)$$

$$=2P_1V_1+\frac{1}{3}P_1V_1=\frac{7}{3}P_1V_1$$

So, option (c) is correct.

Heat supplied to the gas,

$$\Delta Q=W_g+\Delta U$$

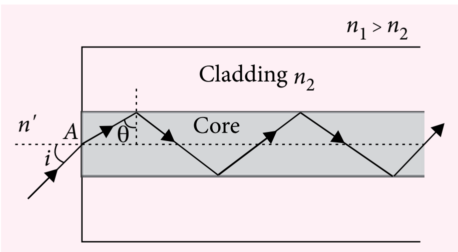
$$=\frac{7}{3}P_1V_1+\frac{3}{2}(P_2V_2-P_1V_1)$$

$$=\frac{7}{3}P_1V_1+\frac{3}{2}\left(\frac{4}{3}P_1\times 3V_1-P_1V_1\right)$$

$$=\frac{7}{3}P_1V_1+\frac{9}{2}P_1V_1=\frac{41}{6}P_1V_1$$

So, option (d) is incorrect.

17. (a, c) : Let optical fiber is placed in a medium of refractive index n'.



Using Snell's law at point A,

$n'\sin i=n_1\sin(90°-\theta)=n_1\cos\theta$ ...(i)

Here for $i=i_m$, $\theta=C$ and $\sin C=\dfrac{n_2}{n_1}$

From eqn. (i)

$n'\sin i_m=n_1\cos C=n_1(1-\sin^2 C)$

$$=n_1\sqrt{1-\frac{n_2^2}{n_1^2}}=\sqrt{n_1^2-n_2^2}$$

$$\therefore\quad \sin i_m=\text{NA}=\frac{\sqrt{n_1^2-n_2^2}}{n'}$$

For S_1, $n_1=\dfrac{\sqrt{45}}{4}$, $n_2=\dfrac{3}{2}$

For S_2, $n_1=\dfrac{8}{5}$, $n_2=\dfrac{7}{5}$

(a) : $(\text{NA})_{S_1}=\dfrac{3}{4}\sqrt{\dfrac{45}{16}-\dfrac{9}{4}}$ $\left(\because\; n'=\dfrac{4}{3}\right)$

$$=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$$

$(\text{NA})_{S_2}=\dfrac{3\sqrt{15}}{16}\sqrt{\dfrac{64}{25}-\dfrac{49}{25}}$ $\left(\because\; n'=\dfrac{16}{3\sqrt{15}}\right)$

$$=\frac{3\sqrt{15}}{16}\times\frac{\sqrt{15}}{5}=\frac{9}{16}=(\text{NA})_{S_1}$$

(b) : $(\text{NA})_{S_1}=\dfrac{\sqrt{15}}{6}\times\dfrac{3}{4}=\dfrac{\sqrt{15}}{8}$ $\left(\because\; n'=\dfrac{6}{\sqrt{15}}\right)$

$(\text{NA})_{S_2}=\dfrac{3}{4}\times\dfrac{\sqrt{15}}{5}\neq(\text{NA})_{S_1}$ $\left(\because\; n'=\dfrac{4}{3}\right)$

(c) : $(\text{NA})_{S_1}=1\times\dfrac{3}{4}=\dfrac{3}{4}$ $(\because\; n'=1)$

$(\text{NA})_{S_2}=\dfrac{\sqrt{15}}{4}\times\dfrac{\sqrt{15}}{5}=\dfrac{3}{4}=(\text{NA})_{S_1}$ $\left(\because\; n'=\dfrac{4}{\sqrt{15}}\right)$

(d) : $(\text{NA})_{S_1}=1\times\dfrac{3}{4}=\dfrac{3}{4}$ $(\because\; n'=1)$

$(\text{NA})_{S_2}=\dfrac{3}{4}\times\dfrac{\sqrt{15}}{5}\neq(\text{NA})_{S_1}$ $\left(\because\; n'=\dfrac{4}{3}\right)$

18. (d) : Given $\text{NA}_2<\text{NA}_1\Rightarrow i_{m_2}<i_{m_1}$

For total internal reflection to take place in both structures, the numerical aperture should be least one for the combined structure. Hence, correct option is (d).

19. (a, d)

20. (a, c)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. A person is painting a ceiling, and a drop of paint falls from the brush onto an operating incandescent lightbulb. The bulb breaks. Why?

– Puneet Shah (Gujrat)

Ans. The glass envelope of an incandescent lightbulb receives energy on the inside surface by electromagnetic radiation from the very hot filament. In addition, because the bulb contains gas, the glass envelope receives energy by matter transfer related to the movement of the hot gas near the filament to the colder glass. Thus, the glass can become very hot. If a drop of relatively cold paint falls onto the glass, that portion of the glass envelope suddenly becomes colder than the other portions, and the contraction of this region can cause thermal stresses that might break the glass.

Q2. Light exhibits Doppler effect. The equation for the Doppler effect for light is not the same equation as that for sound. Why would the equation be different?

– Dinesh Gupta (New Delhi)

Ans. In general, for waves requiring a medium, the speeds of the source and observer can be separately measured with respect to a third entity, the medium. In the Doppler effect for sound, these two speeds are that of the source and that of the observer relative to the air. Because light does not require a medium, no third entity is required. Thus, we cannot identify separate speeds for the source and observer, only their relative speed can be identified. As a result, a different equation must be used, one that contains only this single speed. The appropriate equation for the Doppler effect for light is $\upsilon' = \upsilon\sqrt{\frac{c-v}{c+v}}$, where v is the relative speed between the source and the observer, c is the speed of light, υ' is the frequency of light detected by the observer, and υ is the frequency emitted by the source. This equation can be generated from the laws of relativity.

Q3. A large amount of dust is in the interplanetary space in the solar system. Although this dust can theoretically have a variety of sizes, from molecular size upward, very little of it is smaller than about 0.2 μm in our solar system. Why?

– Riya Khan (UP)

Ans. Dust particles in the solar system are subject to two forces, the gravitational force toward the Sun, and the force from radiation pressure, which is away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the mass of the particle. The radiation pressure is proportional to the square of the radius because it depends on the planar cross section of the particle. For large particles, the gravitational force is larger than the force from radiation pressure. For small particles, less than about 0.2 μm, the larger force from radiation pressures sweeps these particles out of the solar system.

Q4. What is the difference between neutral and ground in an electrical connection?

– Yogesh Yadav (Bihar)

Ans. Neutral is a circuit conductor that may carry current in normal operation, and is connected to ground at the main electrical panel. As its name suggests, the ground wire is connected to the ground, the neutral is not. The ground wire ensures that in case of a leakage, an appliance does not acquire a voltage that might cause injury. It appears that there is no need of separate identification of the live and neutral terminals when you are dealing with a single phase of alternating current. You just need an incoming wire and an outgoing wire to complete the circuit. But many homes and establishments are served by three phases. In that case you would need three separate pairs of electric cables, in other words a total of six cables. On the other hand, if you make one of the cables as a common neutral you would need only four cables, resulting in great saving. You would have noticed therefore that in your switchboard serving a three-phase supply, there are four cables coming in and one of them, called the neutral, is connected to all your appliances and light fixtures.

Class XI

JEE

Maximize your chance of success, and high rank in JEE (Main and Advanced) / BITSAT by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

ACCELERATED LEARNING SERIES

Unit 1 PHYSICS AND MEASUREMENT

PHYSICAL QUANTITIES

All the quantities in terms of which laws of physics can be described and which can be measured are known as physical quantities.

Types of Physical Quantities

There are two types of physical quantities :

- **Fundamental quantities :** Those physical quantities which do not depend upon any other quantity are known as fundamental quantities or base quantities. There are seven fundamental or base quantities in SI system. They are Length, Mass, Time, Electric current, Thermodynamic temperature, Amount of substance and Luminous intensity.
- **Derived quantities :** Those physical quantities which are derived from the fundamental quantities are known as derived quantities.

$$e.g.\ \text{Speed} = \frac{\text{distance}}{\text{time}}$$

UNITS

Unit : Measurement of any physical quantity involves comparison with a certain basic arbitrarily chosen, internationally accepted reference standard known as unit.

The result of a measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit. In general,

Measure of a physical quantity

$$= \text{numerical value } (n) \times \text{unit } (u)$$

KEY POINT

- The magnitude of a physical quantity and its unit are inversely proportional to each other. Larger the unit, smaller the magnitude.

Characteristics of a Standard Unit

A unit selected for measuring a physical quantity should fulfill the following requirements :

- It should be of suitable size.
- It should be well defined.
- It should be easily accessible.
- It should be easily reproducible at all places.
- It should not change with time.
- It should not change with change in its physical conditions like temperature, pressure etc.

Fundamental and Derived Units

- **Fundamental units :** The units of fundamental or base quantities are known as fundamental or base units.
- **Derived units :** The units of derived quantities are known as derived units.

System of Units

- A complete set of units having, both the base units and derived units is known as system of units.

KEY POINT

- In computing any physical quantity, the units for derived quantities involved in the relationship(s) are treated as though they were algebraic quantities till the desired units are obtained.

Different Types of System of Units

- **CGS system :** In this system centimetre, gram and second are the fundamental units of length, mass and time respectively. It is a metric system of units. It is also known as Gaussian system of units.
- **FPS system :** In this system foot, pound and second are the fundamental units of length, mass and time respectively. It is not a metric system of units. It is also known as British system of units.
- **MKS system :** In this system metre, kilogram and second are the fundamental units of length, mass and time respectively. It is also a metric system of units.
- **International System of Units (SI) :** The system of units which is at present internationally accepted for measurement is the Système Internationale d' Unites (French for International System of Units), abbreviated as SI. The SI, with standard scheme of symbols, units and abbreviations, was developed and recommended by General Conference on Weights and Measures in 1971 in France for international usage in scientific, technical, industrial and commercial work. It is based on the seven fundamental units or base units and two supplementary units.

➢ Seven base quantities with their units are as shown in the table.

Base quantity	SI Units	
	Name	**Symbol**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

➢ The two supplementary units in SI system are :
Radian (rad) : It is defined as the plane angle subtended at the centre of circle, by an arc of the circle equal in length to its radius.
Steradian (sr) : It is defined as the solid angle subtended at the centre of a sphere by an area of the sphere equal to square of its radius.

Advantages of SI

The main advantages of SI over the other systems of units are the following :

- SI is a coherent system of units.
- SI is a rational system of units.
- SI is an absolute system of units.
- SI is a metric system.

Practical Units of Length, Mass and Time

Some practical units of length, mass and time are as shown in the table.

Practical Units of Length	Practical Units of Mass	Practical Units of Time
1 astronomical unit = 1 AU = 1.496×10^{11} m (average distance of the sun from the earth)	1 tonne = 10^3 kg	**Solar day :** It is the time interval between two successive passages of the sun across the meridian. 1 Solar day = 86400 s
1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^8 m s^{-1} in 1 year)	1 quintal = 10^2 kg	**Sidereal day :** It is the time interval between two successive passages of a fixed star across the meridian.
1 parsec = 3.08×10^{16} m (parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)	1 pound = 0.4536 kg	**Solar year or year :** It is the time taken by the earth to complete one revolution around the sun in its orbit. 1 solar year = 365.25 average solar days = 366.25 sidereal days
1 micron = 1 μm = 10^{-6} m	1 atomic mass unit = 1 u = 1.66×10^{-27} kg	**Lunar month :** It is the time taken by moon to complete one revolution around the earth in its orbit. 1 lunar month = 29.53 days

1 angstrom = 1 Å = 10^{-10} m	1 Chandrasekhar limit = 1.4 times the mass of sun	**Leap year :** It is that year in which the month of February is of 29 days.
1 fermi = 1 f = 10^{-15} m		1 shake = 10^{-8} s.

SOME CONVERSION FACTORS

Length	1 m = 100 cm 1 km = 1000 m 1 km = 0.6215 mi 1 mi = 1.609 km 1 m = 1.0936 yd = 3.281 ft = 39.37 in 1 in = 2.54 cm 1 ft = 12 in = 30.48 cm 1 yd = 3 ft = 91.44 cm 1 Å = 0.1 nm
Area	1 m^2 = 10^4 cm^2 1 km^2 = 0.3861 mi^2 = 247.1 acres 1 in^2 = 6.4516 cm^2 1 ft^2 = 9.29×10^{-2} m^2 1 m^2 = 10.76 ft^2 1 acre = 43,560 ft^2 1 mi^2 = 640 acres = 2.590 km^2
Volume	1 m^3 = 10^6 cm^3 1 L = 1000 cm^3 = 10^{-3} m^3 1 gal = 3.786 L 1 in^3 = 16.39 cm^3 1 ft^3 = 1728 in^3 = 28.32 L = 2.832×10^4 cm^3
Mass	1 kg = 1000 g 1 tonne = 1000 kg = 1 Mg 1 slug = 14.59 kg 1 kg = 6.852×10^{-2} slug 1 u = 931.50 MeV/c^2
Density	1 g cm^{-3} = 1000 kg m^{-3} = 1 kg L^{-1}
Time	1 min = 60 s 1 h = 60 min = 3.6 ks 1 d = 24 h = 1440 min = 86.4 ks 1 y = 365.25 d = 31.56 Ms
Speed	1 km h^{-1} = 0.2778 m s^{-1} = 0.6215 mi h^{-1} 1 mi h^{-1} = 0.4470 m s^{-1} = 1.609 km h^{-1} 1 mi h^{-1} = 1.467 ft s^{-1}
Angle and angular speed	π rad = 180° 1 rad = 57.30° 1° = 1.745×10^{-2} rad 1 rev min^{-1} = 0.1047 rad s^{-1} 1 rad s^{-1} = 9.549 rev min^{-1}

Force	1 N = 0.2248 lbf = 10^5 dyn 1 lbf = 4.4482 N 1 kgf = 2.2046 lbf
Pressure	1 Pa = 1 N m^{-2} 1 bar = 100 kPa 1 atm = 101.325 kPa = 1.01325 bar 1 atm = 14.7 lbf in^{-2} = 760 mm Hg 1 lbf in^{-2} = 6.895 kPa 1 torr = 1 mm Hg = 133.32 Pa
Energy	1 kW h = 3.6 MJ 1 cal = 4.186 J 1 ft lbf = 1.356 J = 1.286×10^{-3} Btu 1 L atm = 101.325 J 1 L atm = 24.217 cal 1 Btu (British thermal unit) = 252 cal = 1054.35 J 1 eV = 1.602×10^{-19} J 1 u c^2 = 931.50 MeV 1 erg = 10^{-7} J
Power	1 W = 1 J s^{-1} 1 horsepower (hp) = 745.7 W 1 Btu min^{-1} = 17.58 W 1 W = 1.341×10^{-3} hp
Magnetic field	1 G = 10^{-4} T 1 T = 1 Wb m^{-2} = 10^4 G

ACCURACY AND PRECISION OF MEASURING INSTRUMENTS

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument.

Precision tells us to what resolution or limit the quantity is measured by a measuring instrument.

Precision is determined by the least count of the measuring instrument. Smaller the least count, greater is the precision.

ERRORS IN MEASUREMENT

The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is known as error. Every calculated quantity which is based on measured values, also has an error.

The difference in the measured value and the true value of a quantity is known as error in measurement. In general, the errors in measurement can be broadly classified as :

- **Systematic errors :** Systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are :
  - **Instrumental errors :** These arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument etc.
  - Imperfection in experimental technique or procedure.
  - **Personal errors :** These arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

  Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.
- **Random errors :** Random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredictable fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc), personal (unbiased) errors by the observer taking readings etc.
- **Least count error :** The smallest value that can be measured by the measuring instrument is known as its least count. All the readings or measured values are good only up to this value.

  The least count error is the error associated with the resolution of the instrument. Least count error belongs to the category of random errors but within a limited size, it occurs with both systematic and random errors.

  Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.

SELF CHECK

1. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
   (a) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
   (b) A meter scale.
   (c) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
   (d) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
   (JEE Main 2014)

Absolute Error, Relative Error and Percentage Error

Let a physical quantity a be measured n times. Let the measured values be a_1, a_2, a_3, ... a_n. The arithmetic mean of these values is

$$\text{Arithmetic mean, } a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n}$$

$$a_{\text{mean}} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

Absolute error : The magnitude of the difference between the individual measurement and the true value of the quantity is known as the absolute error of the measurement. It is denoted by $|\Delta a|$.

The errors in the individual measured values from the true value are

$$\Delta a_1 = a_1 - a_{\text{mean}}$$
$$\Delta a_2 = a_2 - a_{\text{mean}}$$
$$\Delta a_3 = a_3 - a_{\text{mean}}$$
$$\ldots \quad \ldots \quad \ldots$$
$$\ldots \quad \ldots \quad \ldots$$
$$\Delta a_n = \Delta a_n - a_{\text{mean}}$$

Δa may be positive in certain cases and negative in some other cases. Absolute error $|\Delta a|$ will always be positive.

Mean absolute error : It is the arithmetic mean of all the absolute errors. It is denoted by Δa_{mean}.

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| \ldots + |\Delta a_n|}{n}$$

$$\Delta a_{\text{mean}} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

The final value of measurement may be written as

$$a = a_{\text{mean}} \pm \Delta a_{\text{mean}}$$
$$a_{\text{mean}} - \Delta a_{\text{mean}} \le a \le a_{\text{mean}} + \Delta a_{\text{mean}}$$

This implies that any measurement of the physical quantity a is likely to lie between

$(a_{\text{mean}} + \Delta a_{\text{mean}})$ and $(a_{\text{mean}} - \Delta a_{\text{mean}})$.

Relative error or fractional error : It is defined as the ratio of mean absolute error to the mean value of the quantity measured. Relative error or fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

Percentage error : When the relative error is expressed in percentage, it is known as percentage error.

Percentage error, $\delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

Combination of Errors

Addition : If $X = A + B$, then the maximum absolute error in X is $\Delta X = \Delta A + \Delta B$

When two quantities are added, the maximum absolute error in the final result is the sum of the absolute errors in the individual quantities.

Subtraction : If $X = A - B$, then the maximum absolute error in X is $\Delta X = \Delta A + \Delta B$

When two quantities are subtracted, the maximum absolute error in the final result is the sum of the absolute errors in the individual quantities.

Multiplication : If $X = AB$, then the maximum relative error in X is

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = \frac{\Delta A}{A} \times 100 + \frac{\Delta B}{B} \times 100$$

When two quantities are multiplied, the maximum relative error in the final result is the sum of the relative errors in the individual quantities multiplied.

Division : If $X = \frac{A}{B}$, then the maximum relative error in X is

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = \frac{\Delta A}{A} \times 100 + \frac{\Delta B}{B} \times 100$$

When two quantities are divided, the maximum relative error in the final result is the sum of the relative errors in the individual quantities divided.

Power : If $X = A^k$, then the maximum relative error in X is

$$\frac{\Delta X}{X} = k\frac{\Delta A}{A}$$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = k\frac{\Delta A}{A} \times 100$$

The maximum relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

In more general form, if $X = \frac{A^p B^q}{C^r D^s}$

Maximum relative error in X is

$$\frac{\Delta X}{X} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C} + s\frac{\Delta D}{D}$$

Maximum percentage error in X is

$$\frac{\Delta X}{X} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100 + s\frac{\Delta D}{D} \times 100$$

SELF CHECK

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. The accuracy in the determination of g is
   (a) 1% (b) 5% (c) 2% (d) 3%
   (JEE Main 2015)

3. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is
   (a) zero (b) 1% (c) 3% (d) 6%
   (AIEEE 2012)

SIGNIFICANT FIGURES

The reliable digits plus the first uncertain digit are known as significant digits or significant figures.

Rules to determine the Significant Figures

- **Rule 1 :** All non-zero digits are significant. *e.g.* 123 has three significant figures.
- **Rule 2 :** All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. *e.g.* 108.09 and 10207 have five significant figures each.

- **Rule 3 :** If the number is less than 1, the zero(s) on the right of decimal point, but to the left of the first non-zero digit are not significant. e.g. 0.0072 has two significant figures.
- **Rule 4 :** The terminal or trailing zero(s) in a number without a decimal point are not significant. *e.g.* 13200 has three significant figures.
- **Rule 5 :** The trailing zero(s) in a number with a decimal point are significant. e.g. 6.500 has four significant figures.

  Note : The power (or exponent) of 10 is irrelevant to the determination of significant figures.

  e.g. 4.100×10^3 has four significant figures.

  The change of units only changes the order of exponent but not the number of significant figures. *e.g.* $2.600 \text{ m} = 2.600 \times 10^2 \text{ cm} = 2.600 \times 10^3 \text{ mm} = 2.600 \times 10^{-3} \text{ km}$. All have four significant figures each.

  The digit 0 conventionally put on the left of a decimal for a number less than 1 is never significant *e.g.* 0.125 has three significant figures.

SELF CHECK

4. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
   (a) 4, 4, 2 (b) 5, 1, 2
   (c) 5, 1, 5 (d) 5, 5, 2. *(AIEEE 2010)*

Rounding Off

The result of computation with approximate numbers, which contains more than one uncertain digit, should be rounded off.

Rules regarding rounding off by convention are as follows :

- **Rule 1 :** If the digit to be dropped is less than 5, then the preceding digit is left unchanged. *e.g.* 7.32 is rounded off to 7.3.
- **Rule 2 :** If the digit to be dropped is more than 5, then the preceding digit is raised by one. *e.g.* 6.78 is rounded off to 6.8.
- **Rule 3 :** If the digit to be dropped is 5 followed by non-zero digit then the preceding digit is raised by one. *e.g.* 16.451 is rounded off to 16.5.
- **Rule 4 :** If the digit to be dropped is 5, then preceding digit is left unchanged, if it is even. *e.g.* 6.25 rounding off to 6.2.
- **Rule 5 :** If the digit to be dropped is 5, then the preceding digit is raised by one, if it is odd. *e.g.* 4.75 is rounded off to 4.8.

KEY POINT

- In measured and computed quantities proper significant figures only should be retained. Rules for determining the number of significant figures, carrying out arithmetic operations with them, and 'rounding off' the uncertain digits must be followed.

Rules for Arithmetic Operation with Significant Figures

Addition/Subtraction : In addition or subtraction the final result should retain as many decimal places as are there in the number with the least decimal places.

e.g. 2.1 m + 1.78 m + 1.246 m = 5.126 m.

The final result should be rounded off to 5.1 m.

Multiplication/Division : In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

e.g. Mass = 4.237 g, volume = 2.51 cm^3

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{4.237 \text{ g}}{2.51 \text{ cm}^3} = 1.68804 \text{ g cm}^{-3}$$

The final result should be round off to 1.69 g cm^{-3}.

DIMENSIONAL ANALYSIS

Dimensions of a physical quantity

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. Dimensions are denoted with square brackets [ ].

Length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd], and amount of substance [mol].

Using the square brackets [ ] around a quantity means that we are dealing with the dimensions of the quantity.

Dimensional Formulae and Dimensional Equations

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is known as the dimensional formula of the given physical quantity.

An equation obtained by equating a physical quantity with its dimensional formula is known as the dimensional equation of the physical quantity.

The dimensional formulae and SI units of physical quantities are as shown in the table.

S. N.	Physical quantity	Relation with other physical quantities	Dimensions	Dimensional formula	SI unit
1.	Area	Length × Breadth	[L] × [L]	$[M^0L^2T^0]$	m^2
2.	Volume	Length × Breadth × Height	[L] × [L] × [L]	$[M^0L^3T^0]$	m^3
3.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$\frac{[M]}{[L^3]}$	$[ML^{-3}T^0]$	kg m^{-3}
4.	Speed/Velocity	$\frac{\text{Distance/Displacement}}{\text{Time}}$	$\frac{[L]}{[T]}$	$[M^0LT^{-1}]$	m s^{-1}
5.	Acceleration	$\frac{\text{Velocity}}{\text{Time}}$	$\frac{[LT^{-1}]}{[T]}$	$[M^0LT^{-2}]$	m s^{-2}
6.	Force	Mass × Acceleration	$[M][LT^{-2}]$	$[MLT^{-2}]$	N (newton)
7.	Impulse	Force × Time	$[MLT^{-2}][T]$	$[MLT^{-1}]$	N s
8.	Work	Force × Distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$	J (joule)
9.	Energy	Work	$[ML^2T^{-2}]$	$[ML^2T^{-2}]$	J
10.	Power	$\frac{\text{Work}}{\text{Time}}$	$\frac{[ML^2T^{-2}]}{[T]}$	$[ML^2T^{-3}]$	W (watt)
11.	Momentum	Mass × Velocity	$[M][LT^{-1}]$	$[MLT^{-1}]$	kg m s^{-1}
12.	Pressure, stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{[MLT^{-2}]}{[L^2]}$	$[ML^{-1}T^{-2}]$	N m^{-2} or Pa (Pascal)
13.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$\frac{[L]}{[L]}$	$[M^0L^0T^0]$	No units
14.	Modulus of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[ML^{-1}T^{-2}]$	N m^{-2}
15.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{[MLT^{-2}]}{[L]}$	$[ML^0T^{-2}]$	N m^{-1}
16.	Specific gravity/ relative density	$\frac{\text{Density of body}}{\text{Density of water at 4°C}}$	$\frac{[ML^{-3}]}{[ML^{-3}]}$	$[M^0L^0T^0]$	No units
17.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{[LT^{-1}]}{[L]}$	$[M^0L^0T^{-1}]$	s^{-1}
18.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{[ML^{-1}T^{-2}]}{[L]}$	$[ML^{-2}T^{-2}]$	N m^{-3}
19.	Pressure energy	Pressure × Volume	$[ML^{-1}T^{-2}][L^3]$	$[ML^2T^{-2}]$	J
20.	Coefficient of viscosity	$\frac{\text{Force}}{\text{Area} \times \text{velocity gradient}}$	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1}T^{-1}]$	Pa s (Pascal second)
21.	Angle, Angular displacement	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{[L]}{[L]}$	$[M^0L^0T^0]$	rad (radian)
22.	Angular velocity	$\frac{\text{Angle}}{\text{Time}}$	$\frac{[M^0L^0T^0]}{[T]}$	$[M^0L^0T^{-1}]$	rad s^{-1}
23.	Angular acceleration	$\frac{\text{Angular velocity}}{\text{Time}}$	$\frac{[T^{-1}]}{[T]}$	$[M^0L^0T^{-2}]$	rad s^{-2}
24.	Radius of gyration	Distance	[L]	$[M^0LT^0]$	m

25.	Moment of inertia	Mass × (Radius of gyration)^2	$[M][L^2]$	$[ML^2T^0]$	kg m^2
26.	Angular momentum	Moment of inertia × Angular velocity	$[ML^2][T^{-1}]$	$[ML^2T^{-1}]$	kg m^2 s^{-1}
27.	Moment of force, moment of couple	Force × Distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$	N m
28.	Torque	Force × Distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$	N m
29.	Angular frequency	2π × Frequency	$[T^{-1}]$	$[M^0L^0T^{-1}]$	rad s^{-1}
30.	Wavelength	Distance	$[L]$	$[M^0LT^0]$	m
31.	Hubble constant	$\dfrac{\text{Recession speed}}{\text{Distance}}$	$\dfrac{[LT^{-1}]}{[L]}$	$[M^0L^0T^{-1}]$	s^{-1}
32.	Intensity of wave	$\dfrac{\text{Energy}}{\text{Time} \times \text{area}}$	$\dfrac{[ML^2T^{-2}]}{[T][L^2]}$	$[ML^0T^{-3}]$	W m^{-2}
33.	Radiation pressure	$\dfrac{\text{Intensity of wave}}{\text{Speed of light}}$	$\dfrac{[ML^0T^{-3}]}{[LT^{-1}]}$	$[ML^{-1}T^{-2}]$	N m^{-2}
34.	Kinetic energy	$\dfrac{1}{2}$ × Mass × (Velocity)^2	$[M][LT^{-1}]^2$	$[ML^2T^{-2}]$	J
35.	Potential energy	Mass × Acceleration due to gravity × Height	$[M][LT^{-2}][L]$	$[ML^2T^{-2}]$	J
36.	Angular impulse	Torque × Time	$[ML^2T^{-2}][T]$	$[ML^2T^{-1}]$	kg m^2 s^{-1}
37.	Gravitational constant	$\dfrac{\text{Force} \times (\text{Distance})^2}{\text{Mass} \times \text{Mass}}$	$\dfrac{[MLT^{-2}][L^2]}{[M][M]}$	$[M^{-1}L^3T^{-2}]$	N m^2 kg^{-2}
38.	Planck's constant	$\dfrac{\text{Energy}}{\text{Frequency}}$	$\dfrac{[ML^2T^{-2}]}{[T^{-1}]}$	$[ML^2T^{-1}]$	J s
39.	Heat capacity, entropy	$\dfrac{\text{Heat energy}}{\text{Temperature}}$	$\dfrac{[ML^2T^{-2}]}{[K]}$	$[ML^2T^{-2}K^{-1}]$	J K^{-1}
40.	Specific heat capacity	$\dfrac{\text{Heat energy}}{\text{Mass} \times \text{Temperature}}$	$\dfrac{[ML^2T^{-2}]}{[M][K]}$	$[M^0L^2T^{-2}K^{-1}]$	J kg^{-1} K^{-1}
41.	Latent heat	$\dfrac{\text{Heat energy}}{\text{Mass}}$	$\dfrac{[ML^2T^{-2}]}{[M]}$	$[M^0L^2T^{-2}]$	J kg^{-1}
42.	Coefficient of thermal conductivity	$\dfrac{\text{Heat energy} \times \text{Thickness}}{\text{Area} \times \text{Temperature} \times \text{Time}}$	$\dfrac{[ML^2T^{-2}][L]}{[L^2][K][T]}$	$[MLT^{-3}K^{-1}]$	W m^{-1} K^{-1}
43.	Bulk modulus	$\dfrac{\text{Volume} \times (\text{Change in pressure})}{\text{Change in volume}}$	$\dfrac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	$[ML^{-1}T^{-2}]$	N m^{-2}
44.	Stefan's constant	$\dfrac{\text{Energy}}{(\text{Area}) \times (\text{time}) \times (\text{temperature})^4}$	$\dfrac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	$[ML^0T^{-3}K^{-4}]$	W m^{-2} K^{-4}
45.	Wien's constant	Wavelength × Temperature	$[L][K]$	$[M^0LT^0K]$	m K
46.	Universal gas constant	$\dfrac{\text{Pressure} \times \text{Volume}}{\text{Mole} \times \text{Temperature}}$	$\dfrac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$	J K^{-1} mol^{-1}
47.	Boltzmann constant	$\dfrac{\text{Universal gas constant}}{\text{Avogadro's number}}$	$\dfrac{[ML^2T^{-2}K^{-1}mol^{-1}]}{[mol^{-1}]}$	$[ML^2T^{-2}K^{-1}]$	J K^{-1}
48.	Charge	Current × Time	$[A][T]$	$[M^0L^0TA]$	C (coulomb)

49.	Current density	$\frac{\text{Current}}{\text{Area}}$	$\frac{[A]}{[L^2]}$	$[M^0L^{-2}T^0A]$	A m^{-2}
50.	Electric potential, EMF, voltage	$\frac{\text{Work}}{\text{Charge}}$	$\frac{[ML^2T^{-2}]}{[AT]}$	$[ML^2T^{-3}A^{-1}]$	V (volt)
51.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$	Ω (ohm)
52.	Capacitance	$\frac{\text{Charge}}{\text{Potential difference}}$	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^4A^2]$	F (farad)
53.	Electric field	$\frac{\text{Electrical force}}{\text{Charge}}$	$\frac{[MLT^{-2}]}{[AT]}$	$[MLT^{-3}A^{-1}]$	N C^{-1}
54.	Electric flux	Electric field × Area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$	N m^2 C^{-1}
55.	Electric dipole moment	Charge × Length	$[AT][L]$	$[M^0LTA]$	C m
56.	Electric field strength or electric intensity	$\frac{\text{Potential difference}}{\text{Distance}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[L]}$	$[MLT^{-3}A^{-1}]$	V m^{-1}
57.	Magnetic field, magnetic flux density, magnetic induction	$\frac{\text{Force}}{\text{Current} \times \text{Length}}$	$\frac{[MLT^{-2}]}{[A][L]}$	$[ML^0T^{-2}A^{-1}]$	T (tesla)
58.	Magnetic flux	Magnetic field × Area	$[ML^0T^{-2}A^{-1}][L^2]$	$[ML^2T^{-2}A^{-1}]$	Wb (weber)
59.	Inductance	$\frac{\text{Magnetic flux}}{\text{Current}}$	$\frac{[ML^2T^{-2}A^{-1}]}{[A]}$	$[ML^2T^{-2}A^{-2}]$	H (henry)
60.	Magnetic dipole moment	Current × Area	$[A][L^2]$	$[M^0L^2T^0A]$	A m^2
61.	Intensity of magnetisation	$\frac{\text{Magnetic moment}}{\text{Volume}}$	$\frac{[L^2A]}{[L^3]}$	$[M^0L^{-1}T^0A]$	A m^{-1}
62.	Permittivity of free space	$\frac{\text{Charge} \times \text{Charge}}{4\pi \times \text{Electric force} \times (\text{Distance})^2}$	$\frac{[AT][AT]}{[MLT^{-2}][L]^2}$	$[M^{-1}L^{-3}T^4A^2]$	C^2 N^{-1} m^{-2}
63.	Permeability of free space	$\frac{2\pi \times \text{Force} \times \text{Distance}}{\text{Current} \times \text{Current} \times \text{Length}}$	$\frac{[MLT^{-2}][L]}{[A][A][L]}$	$[MLT^{-2}A^{-2}]$	T m A^{-1}
64.	Wave number	$\frac{2\pi}{\text{Wavelength}}$	$\frac{1}{[L]}$	$[M^0L^{-1}T^0]$	rad m^{-1}
65.	Radiant flux, radiant power	$\frac{\text{Energy emitted}}{\text{Time}}$	$\frac{[ML^2T^{-2}]}{[T]}$	$[ML^2T^{-3}]$	watt
66.	Power of lens	$(\text{Focal length})^{-1}$	$[L]^{-1}$	$[M^0L^{-1}T^0]$	dioptre
67.	Magnification	$\frac{\text{Size of image}}{\text{Size of object}}$	$\frac{[L]}{[L]}$	$[M^0L^0T^0]$	No units
68.	Fluid flow rate	$\frac{(\pi/8)(\text{Pressure}) \times (\text{Radius})^4}{(\text{Viscosity coefficient}) \times (\text{Length})}$	$\frac{[ML^{-1}T^{-2}][L]^4}{[ML^{-1}T^{-1}][L]}$	$[M^0L^3T^{-1}]$	m^3 s^{-1}
69.	Capacitive reactance	(Angular frequency × Capacitance)$^{-1}$	$[T^{-1}]^{-1}[M^{-1}L^{-2}T^4A^2]^{-1}$	$[ML^2T^{-3}A^{-2}]$	ohm
70.	Inductive reactance	(Angular frequency × Inductance)	$[T^{-1}][ML^2T^{-2}A^{-2}]$	$[ML^2T^{-3}A^{-2}]$	ohm

SELF CHECK

5. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then
(a) $[\in_0] = [M^{-1} L^2 T^{-1} A]$
(b) $[\in_0] = [M^{-1} L^{-3} T^2 A]$
(c) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$
(d) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ *(JEE Main 2013)*

6. The dimension of magnetic field in M, L, T and C (coulomb) is given as
(a) $MT^{-2}C^{-1}$ (b) $MLT^{-1}C^{-1}$
(c) MT^2C^{-2} (d) $MT^{-1}C^{-1}$. *(AIEEE 2008)*

Physical Quantities having Same Dimensional Formulae

S.N.	Physical Quantities	Dimensional Formula
1.	Frequency, angular frequency, angular velocity, velocity gradient	$[M^0L^0T^{-1}]$
2.	Work, internal energy, potential energy, kinetic energy, torque, moment of force	$[ML^2T^{-2}]$
3.	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density	$[ML^{-1}T^{-2}]$
4.	Momentum and impulse	$[MLT^{-1}]$
5.	Acceleration, Acceleration due to gravity, gravitational field intensity	$[M^0LT^{-2}]$
6.	Thrust, force, weight, energy gradient	$[MLT^{-2}]$
7.	Angular momentum and Planck's constant (h)	$[ML^2T^{-1}]$
8.	Surface tension, force gradient, spring constant	$[ML^0T^{-2}]$
9.	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permeability, relative permittivity	$[M^0L^0T^0]$
10.	If P is pressure, V is volume, m is mass, s is specific heat, L is latent heat, ΔT is rise in temperature then PV, mL, $(ms\Delta T)$ all have dimensions of energy	$[ML^2T^{-2}]$
11.	If l is length, g is acceleration due to gravity, m is mass, k is force constant, R is radius of earth, then $\left(\frac{l}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{R}{g}\right)^{1/2}$ all have the dimensions of time.	$[M^0L^0T]$
12.	If L is inductance, R is resistance, C is capacitance then L/R, CR and $\sqrt{LC}$ all have the dimensions of time.	$[M^0L^0T]$

Applications of Dimensional Analysis

The main applications of dimensional analysis are the following :

- **To check the dimensional consistency of equations :** It is based on principle of homogeneity of dimensions which states that the equation is dimensionally correct if the dimensions of the various terms on either side of the equation are the same.
- **To deduce relation among the physical quantities :** If we know the dependence of the physical quantity on the other physical quantities, we can derive a relation among the physical quantities by using the principle of homogeneity of dimensions.
- **To convert one system of unit into another system of unit :** For this, we use the relation

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

where M_1, L_1, T_1 are fundamental units on one system; M_2, L_2, T_2 are fundamental units on the other system, a, b, c are the dimensions of the quantity in mass, length and time, n_1 is numerical value in one system and n_2 is its numerical value in the other system.

Note : This formula is valid only for absolute units and not for gravitational units.

KEY POINT

- Only those physical quantities can be added or subtracted from each other which have the same dimensions.
- A dimensionally consistent equation need not be actually an exact (correct) equation, but a dimensionally wrong or inconsistent equation must be wrong.

ANSWER KEYS (SELF CHECK)

1. (c) 2. (d) 3. (d) 4. (b) 5. (c)
6. (d)

Exam Café

QUESTIONS FOR PRACTICE

1. The radius of the proton is about 10^{-15} m. The radius of the observable universe is 10^{26} m. Identify the distance which is half-way between these two extremes on a logarithmic scale.
(a) 10^{21} m (b) 10^{6} m
(c) 10^{-6} m (d) 10^{0} m

2. What will be the unit of time in that system in which the unit of length is metre, unit of mass is kg and unit of force is kg wt?
(a) $(9.8)^2$ sec (b) 9.8 sec
(c) $\sqrt{9.8}$ sec (d) $\frac{1}{\sqrt{9.8}}$ sec

3. The number of particles given by $n = -D\frac{n_2 - n_1}{x_2 - x_1}$ are crossing a unit area perpendicular to x-axis in unit time, where n_1 and n_2 are the number of particles per unit volume for the values x_1 and x_2 of x respectively. Then the dimensional formula of diffusion constant D is
(a) $[M^0LT^0]$ (b) $[M^0L^2T^{-4}]$
(c) $[M^0LT^{-3}]$ (d) $[M^0L^2T^{-1}]$

4. What is the unit of k in the relation $U = \frac{ky}{y^2 + a^2}$ where U represents the potential energy, y represents the displacement and a represents the maximum displacement *i.e.*, amplitude?
(a) m s^{-1} (b) m s (c) J m (d) J s^{-1}

5. The mass and density of a solid sphere are measured to be (12.4 ± 0.1) kg and (4.6 ± 0.2) kg m^{-3}. Calculate the volume of the sphere with error limits (in m^3).
(a) 2.4 ± 0.14 (b) 2.7 ± 0.18
(c) 2.6 ± 0.18 (d) 2.7 ± 0.14

6. If the error in the measurement of momentum of a particle is (+100%), then the error in the measurement of kinetic energy is
(a) 100% (b) 200% (c) 300% (d) 400%

7. The dimensional representation of specific resistance in terms of charge Q is
(a) $[ML^3T^{-1}Q^{-2}]$ (b) $[ML^2T^{-2}Q^2]$
(c) $[MLT^{-2}Q^{-1}]$ (d) $[ML^2T^{-2}Q^{-1}]$

8. It T denotes the surface tension and R, the radius of a capillary tube, then the dimensions of T/R are the same as that of
(a) pressure (b) energy
(c) force (d) none of these

9. If units of two systems of measurement are in the ratio 2 : 3, then the ratio of units of angular momentum in those two system is
(a) 2 : 3 (b) 9 : 4 (c) 4 : 9 (d) 1 : 1

10. The relative density of a metal may be found by hanging a block of the metal from a spring balance and noting that in air, the balance reads (5.00 ± 0.05) N, while in water it reads (4.00 ± 0.05) N. the relative density would be quoted as
(a) 5.00 ± 0.05 (b) 5.00 ± 0.10
(c) 5.00 ± 6% (d) 5.00 ± 11%

11. The time taken by a vehicle to go from one station to the other is 100 seconds. It is recorded with a stop watch having least count of 1 second. How many significant figures are there in t = 100 s?
(a) 2 (b) 1 (c) 3 (d) 4

12. If velocity (v), acceleration (a) and force (F) are taken as fundamental quantities, the dimensions of Young's modulus (Y) would be
(a) Fa^2v^{-2} (b) Fa^2v^{-3} (c) Fa^2v^{-4} (d) Fa^2v^{-5}

13. The dimensional formula for acceleration, velocity and length are $\alpha\beta^{-2}$, $\alpha\beta^{-1}$ and $\alpha\gamma$. What is the dimensional formula for the coefficient of friction?
(a) $\alpha\beta\gamma$ (b) $\alpha^{-1}\beta^0\gamma^0$
(c) $\alpha^0\beta^{-1}\gamma^0$ (d) $\alpha^0\beta^0\gamma^{-1}$

14. In the measurement of a physical quantity $X = \frac{A^2B}{C^{1/3}D^3}$, the percentage errors introduced in the measurements of the quantities A, B, C and D are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by
(a) A (b) B (c) C (d) D

15. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
(a) weber (Wb) (b) Wb/m^2
(c) henry (H) (d) H/m^2

16. The dimensions of the quantity namely $\frac{\mu_0 ce^2}{2\hbar}$ where μ_0 – permeability of free space, c – velocity

of light, e – electronic charge and $\hbar = \frac{h}{2\pi}$, h being Planck's constant, is

(a) $[M^0LT]$ (b) $[M^0L^0T]$
(c) $[M^0L^0T^0]$ (d) $[M^{-2}L^{-1}T^{-3}]$

17. The dimensions of σb^4 are (where σ = Stefan's constant and b = Wien's constant)
(a) $[M^0L^0T^0]$ (b) $[ML^4T^{-3}]$
(c) $[ML^{-2}T]$ (d) $[ML^6T^{-3}]$

18. If 3.8×10^{-6} is added to 4.2×10^{-5} giving due regard to significant figures, then the result will be
(a) 4.58×10^{-5} (b) 4.6×10^{-5}
(c) 4.5×10^{-5} (d) 4.7×10^{-5}

19. The circular scale of a screw gauge has 100 equal divisions. When it is given 4 complete rotations, it moves through 2 mm. The L.C. of screw gauge is
(a) 0.005 cm (b) 0.0005 cm
(c) 0.001 cm (d) 0.0001 cm

20. Solar constant may be defined as the amount of solar energy received per cm^2 per minute. The dimensions of solar constant is
(a) $[ML^2T^{-3}]$ (b) $[ML^0T^{-1}]$
(c) $[ML^0T^{-2}]$ (d) $[ML^0T^{-3}]$

21. Out of the following pairs which one does not have identical dimensions?
(a) Moment of inertia and moment of a force
(b) Work and torque
(c) Angular momentum and Planck's constant
(d) Impulse and momentum

22. If dimensions of length are expressed as $G^x c^y h^z$, where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(a) $x = \frac{1}{2}, y = \frac{1}{2}, z = \frac{1}{2}$

(b) $x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

(c) $x = \frac{3}{2}, y = \frac{1}{2}, z = \frac{1}{2}$

(d) $x = -\frac{1}{2}, y = -\frac{1}{2}, z = -\frac{1}{2}$

23. The dimensions of $\frac{a}{b}$ in the equation $P = \frac{a - t^2}{bx}$, where P is pressure, x is distance and t is time are
(a) $[M^2LT^{-3}]$ (b) $[ML^0T^{-2}]$
(c) $[ML^3T^{-1}]$ (d) $[M^0LT^{-3}]$

24. The mass of a block is 87.2 g and its volume is 25 cm^3. Its density upto correct significant figures is
(a) 3.488 g cm^{-3} (b) 3.5 g cm^{-3}
(c) 3.48 g cm^{-3} (d) 3.4 g cm^{-3}

25. A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion.

(1) $y = a\sin\frac{2\pi t}{T}$ (2) $y = a\sin vt$

(3) $y = \frac{a}{T}\sin\frac{t}{a}$

(4) $y = (a\sqrt{2})\left(\sin\frac{2\pi t}{T} + \cos\frac{2\pi t}{T}\right)$

where a is the maximum displacement of the particle, v is the speed of the particle, T is the time period of motion. Then dimensionally
(a) 1 and 2 are wrong
(b) 2 and 3 are wrong
(c) 3 and 4 are wrong
(d) 4 and 1 are wrong

26. The position x of a particle at time t is given by

$$x = \frac{V_0}{a}(1 - e^{-at})$$

where V_0 is a constant and $a > 0$.
The dimensions of V_0 and a are
(a) $[M^0LT^{-1}]$ and $[M^0L^0T^{-1}]$
(b) $[M^0LT^0]$ and $[M^0LT^{-1}]$
(c) $[M^0LT^{-1}]$ and $[MLT^{-2}]$
(d) $[M^0LT^{-1}]$ and $[M^0LT]$

27. Which of the following quantities has the SI unit kg m^2 s^{-3} A^{-2}?
(a) Resistance (b) Inductance
(c) Capacitance (d) Magnetic flux

28. Velocity v is given by $v = at^2 + bt + c$, where t is time. What are the dimensions of a, b and c respectively?
(a) $[LT^{-3}]$, $[LT^{-2}]$ and $[LT^{-1}]$
(b) $[LT^{-1}]$, $[LT^{-2}]$ and $[LT^{-3}]$
(c) $[LT^{-2}]$, $[LT^{-3}]$ and $[LT^{-1}]$
(d) $[LT^{-1}]$, $[LT^{-3}]$ and $[LT^{-2}]$

29. Percentage error in the measurement of mass and speed are 2% and 3%, respectively. The percentage error in the estimation of kinetic energy obtained by measuring mass and speed will be
(a) 12% (b) 10% (c) 2% (d) 8%

30. In the following question, match Column-I and Column-II and select the correct match out of the four given choices.

Column-I		Column-II
A. Specific heat	P.	$[M^1L^2T^{-2}K^{-1}]$
B. Boltzmann constant	Q.	$[M^0L^1T^0K^1]$
C. Wien's constant	R.	$[M^1L^0T^{-3}K^{-4}]$
D. Stefan's constant	S.	$[M^0L^2T^{-2}K^{-1}]$

(a) A-S; B-R; C-Q; D-P (b) A-S; B-P; C-Q; D-R
(c) A-Q; B-R; C-P; D-S (d) A-P; B-Q; C-R; D-S

SOLUTIONS

1. (b): $r_1 = 10^{-15}$ m, $r_2 = 10^{26}$ m.

$$\log r = \frac{1}{2}[\log 10^{-15} + \log 10^{26}]$$
$$= \frac{1}{2}[-15+26] = 5.5 \approx 6 \quad \text{or,} \quad r = 10^6 \text{ m}$$

2. (d): We know $[F] = [MLT^{-2}]$

$$\therefore \quad T^2 = \frac{ML}{F} = \frac{1\,\text{kg}\times 1\,\text{m}}{1\,\text{kg-wt}} = \frac{1\,\text{kg}\times 1\,\text{m}}{9.8\,\text{N}}$$

or $\quad T = \frac{1}{\sqrt{9.8}}$ sec

3. (d): From the given relation, $D = -\frac{n(x_2-x_1)}{n_2-n_1}$

Here $[n] = \left[\frac{1}{\text{area}\times\text{time}}\right] = \frac{1}{[L^2T]} = [L^{-2}T^{-1}]$

$[x_2-x_1] = [L]$ and $[n_2-n_1] = \left[\frac{1}{\text{volume}}\right] = \left[\frac{1}{L^3}\right] = [L^{-3}]$

So, $[D] = \frac{[L^{-2}T^{-1}L]}{[L^{-3}]} = [L^2T^{-1}]$

4. (c) : The right hand side of the given relation is basically $\frac{k}{\text{metre}}$. But, since the left hand side is joule, therefore k should be J m.

5. (d): Here, $m \pm \Delta m = (12.4 \pm 0.1)$ kg
and $\rho \pm \Delta\rho = (4.6 \pm 0.2)\text{kg m}^{-3}$

$$\text{Volume } V = \frac{m}{\rho} = \frac{12.4}{4.6} = 2.69\text{ m}^3 = 2.7\text{ m}^3$$

(rounding off to one decimal place)

Now, $\frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$

or $\quad \Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)\times V$

$$= \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right)\times 2.7 = \pm\, 0.14$$

$\therefore \quad V \pm \Delta V = (2.7 \pm 0.14)\text{ m}^3$

6. (c) : Since error in measurement of momentum is +100%

$$\therefore \quad P_1 = P,\ P_2 = 2P \Rightarrow K_1 = \frac{P^2}{2m},\ K_2 = \frac{(2P)^2}{2m}$$

$$\%\text{ error in } K = \left(\frac{K_2-K_1}{K_1}\right)\times 100 = \left(\frac{4-1}{1}\right)\times 100$$
$$= 300\%$$

7. (a)

8. (a)

9. (c) : $\frac{u_1}{u_2} = \frac{M_1L_1^2T_1^{-1}}{M_2L_2^2T_2^{-1}} = \left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-1}$

$$= \frac{2}{3}\times\left(\frac{2}{3}\right)^2\left(\frac{2}{3}\right)^{-1} = \frac{4}{9}$$

10. (d): Relative density $(\rho) = \frac{\text{Weight in air}}{\text{Loss of weight in water}}$

$$= \frac{5.00}{5.00-4.00} = \frac{5.00}{5.00-4.00} = 5.00$$

$$\frac{\Delta\rho}{\rho}\times 100 = \left(\frac{0.05}{5.00} + \frac{0.05+0.05}{1.00}\right)\times 100 = 11\%$$

$\therefore \quad$ Relative density = 5.00 ± 11%

11. (c) : As the measured time is 100 s, therefore number of significant figures = 3.

12. (c) : Let $Y = v^x a^y F^z$

$\therefore \quad [ML^{-1}T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [MLT^{-2}]^z$
$= M^z L^{x+y+z} T^{-x-2y-2z}$

Equating the powers of M, L and T, we get
$z = 1,\ x + y + z = -1;\ -x - 2y - 2z = -2$
Solving, $y = 2,\ z = 1,\ x = -4$
$\therefore \quad Y = v^{-4}a^2F^1 = Fa^2v^{-4}$

13. (d): Here, $[a] = LT^{-2} = \alpha\beta^{-2}$, $[v] = LT^{-1} = \alpha\beta^{-1}$,
$\therefore \quad \alpha = L,\ \beta = T$

As $\quad [L] = \alpha\gamma \quad \therefore \quad \gamma = \frac{L}{\alpha} = \frac{L}{L} = 1$

Coefficient of friction $\mu = \frac{F}{R} = M^0L^0T^0$

Check all the four given expression and find which one is dimensionless.
$\alpha^0\beta^0\gamma^{-1} = L^0T^0(1)^{-1} = 1$, which is dimensionless.

14. (c) : Given $X = \frac{A^2B}{C^{1/3}D^3}$

Taking logarithm of both sides, we have

$\log X = 2\log A + \log B - \frac{1}{3}\log C - 3\log D$

Partially differentiating, we have

$$\frac{\Delta X}{X} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{1}{3}\frac{\Delta C}{C} - 3\frac{\Delta D}{D}$$

Percentage error in A = $2\frac{\Delta A}{A} = 2\times 2\% = 4\%$

Percentage error in $B = \frac{\Delta B}{B} = 2\%$

Percentage error in $C = \frac{1}{3}\frac{\Delta C}{C} = \frac{1}{3} \times 4\% = \frac{4}{3}\%$

Percentage error in $D = 3\frac{\Delta D}{D} = 3 \times 5\% = 15\%$

We find that the minimum percentage error is contributed by C. Hence the correct choice is (c).

15. (c)

16. (c) : The quantity is $\frac{\mu_0 ce^2}{2\hbar} = \frac{\pi\mu_0 ce^2}{h}$

$\because \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad \text{or} \quad c^2 = \frac{1}{\mu_0\varepsilon_0}$

$\therefore \quad c\mu_0 = \frac{1}{c\varepsilon_0} \quad \therefore \quad \frac{\pi\mu_0 ce^2}{h} = \frac{\pi e^2}{c\varepsilon_0 h}$

$\because \quad F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \Rightarrow \quad \frac{e^2}{\varepsilon_0} = Fr^2\, 4\pi$

$\therefore \quad \frac{\pi e^2}{c\varepsilon_0 h} = \frac{Fr^2}{ch} \times 4\pi^2$

The dimensions of

$$\frac{Fr^2}{ch} = \frac{[MLT^{-2}][L^2]}{[LT^{-1}][ML^2T^{-1}]} = \frac{[ML^3T^{-2}]}{[ML^3T^{-2}]} = [M^0L^0T^0]$$

17. (b): $\lambda_m T = b \quad \text{or} \quad b^4 = \lambda_m^4 T^4$

and $\frac{\text{Energy}}{\text{Area} \times \text{time}} = \sigma T^4 \quad \text{or} \quad \sigma = \frac{\text{energy}}{(\text{area} \times \text{time})T^4}$

or $\sigma b^4 = \left(\frac{\text{Energy}}{\text{area} \times \text{time}}\right)\lambda_m^4$

or $[\sigma b^4] = \frac{[ML^2T^{-2}]}{[L^2][T]}[L^4] = [ML^4T^{-3}]$

18. (b): We will use the general rule of addition by making the powers same.

i.e., we will add 3.8×10^{-6} and 42×10^{-6}, we get

$= 45.8 \times 10^{-6} = 4.58 \times 10^{-5}$.

As least number of significant figures in given values are 2, so we round off the result to 4.6×10^{-5}.

19. (b): Pitch $= \frac{2}{4}$ mm $= \frac{1}{2}$ mm $= \frac{1}{20}$ cm.

L.C. $= \frac{\text{Pitch}}{100} = \frac{1}{2000}$ cm = 0.0005 cm.

20. (d): Solar constant $S = \frac{\text{solar energy}}{\text{area} \times \text{time}} = \frac{[ML^2T^{-2}]}{[L^2][T]}$

$= [ML^0T^{-3}]$.

21. (a): Moment of inertia $(I) = mr^2$

$\therefore \quad [I] = [ML^2]$

Moment of force $(\vec{\tau}) = \vec{r} \times \vec{F}$

$\therefore \quad [\tau] = [r][F] = [L][MLT^{-2}] \text{ or } [\tau] = [ML^2T^{-2}]$

Moment of inertia and moment of a force do not have identical dimensions.

22. (b): Let $L = kG^x c^y h^z$

where k is a dimensionless constant of proportionality.

Equating dimensions on both sides, we get

$[M^0LT^0] = [M^{-1}L^3T^{-2}]^x[LT^{-1}]^y[ML^2T^{-1}]^z$

$= [M^{-x+z}L^{3x+y+2z}T^{-2x-y-z}]$

Applying principle of homogeneity of dimensions, we get

$-x + z = 0$ ...(i)

$3x + y + 2z = 1$ ...(ii)

$-2x - y - z = 0$ ...(iii)

Solving equations (i), (ii) and (iii), we get

$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

23. (b) : $P = \frac{a - t^2}{bx}$

Dimensions of $a = [T^2]$, as t^2 is subtracted from a.

From $[P] = \left[\frac{a - t^2}{bx}\right] = \left[\frac{t^2}{bx}\right]$, $[b] = \frac{[t^2]}{[Px]}$

$\therefore \quad [b] = \frac{[T^2]}{[ML^{-1}T^{-2}][L]} = [M^{-1}L^0T^4]$

$\left[\frac{a}{b}\right] = \frac{[T^2]}{[M^{-1}L^0T^4]} = [ML^0T^{-2}]$

24. (b) **25. (b)** **26. (a)**

27. (a): kg m^2 s^{-3} A^{-2} = $[ML^2T^{-3}A^{-2}]$

$= \frac{[ML^2T^{-2}]}{[A^2T]} = \frac{[ML^2T^{-2}]}{[AT][A]}$

$= \frac{\text{Work}}{\text{Charge} \times \text{Current}} = \frac{\text{Voltage}}{\text{Current}} = \text{Resistance}$

28. (a)

29. (d): Kinetic energy, $K = \frac{1}{2}mv^2$

Fractional error in kinetic energy $\frac{\Delta K}{K} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$

Percentage error in estimation of kinetic energy

$\frac{\Delta K}{K} \times 100 = \left[\frac{\Delta m}{m} + 2\frac{\Delta v}{v}\right] \times 100$

$= 2\% + 2 \times 3\% = 2\% + 6\% = 8\%$

30. (b): As it is known from the dimensions of the quantities involved, choice (b) is correct.

BRAIN MAP

MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

Magnetic resonance imaging (MRI), a medical imaging technique used in radiology to investigate the anatomy and physiology of the body, use magnetic fields and radio waves to form images of the body. It is used for diagnosis, staging of disease and for follow-up without exposure to ionizing radiation.

Magnetic Force

- Magnetic force on a moving charge,
  $\vec{F}_m = q(\vec{v} \times \vec{B})$
  $F_m = qvB \sin\theta$
  θ is the angle between $\vec{v}$ and $\vec{B}$.
- Lorentz force, $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$
- Force on a current carrying conductor in a uniform magnetic field (B).
  $\vec{F} = I\vec{l} \times \vec{B} = IlB\sin\theta\,\hat{n}$
  This force acts on the COM of rod.

Charged Particle in Uniform Magnetic Field

- If $\theta = 0°$ or $180°$, path is a straight line.
- If $\theta = 90°$, path is circle.
  Radius of circle,
  $$r = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB} = \frac{\sqrt{2qVm}}{qB}$$
  Time period,
  $$T = \frac{2\pi m}{qB},\ \omega = \frac{qB}{m},\ \upsilon = \frac{qB}{2\pi m}$$
- For any other angle path is helix.
  Radius, $r = \dfrac{mv\sin\theta}{qB}$
  Time period, $T = \dfrac{2\pi m}{qB} = \dfrac{1}{\upsilon}$
  Pitch of helical path $p = (v\cos\theta)T = \dfrac{2\pi mv\cos\theta}{qB}$

Cyclotron

- It is machine to accelerate charged particles or ions to high energies.
- Cyclotron frequency,
  $\upsilon_c = \dfrac{qB}{2\pi m}$

Biot-Savart Law

- Magnetic field due to a current carrying element,
  $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(d\vec{l} \times \vec{r})}{r^3}$$

Applications of Biot-Savart Law

- Magnetic field due to a straight wire of finite length,
  $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{R}(\sin\alpha + \sin\beta)$
- Magnetic field due to a straight wire of infinite length,
  $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{R}$
- Magnetic field on the axis of a circular loop
  $B = \dfrac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$
- Magnetic field at the centre of a circular loop
  $B = \dfrac{\mu_0 NI}{2R}$
- Magnetic field at the centre due to arc of circle
  $B = \left(\dfrac{\theta}{2\pi}\right)\left(\dfrac{\mu_0 NI}{2R}\right)$
- Magnetic field on the axis of a solenoid,
  $B = \dfrac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$
- Magnetic field at centre (on the axis) of a long solenoid
  $B = \mu_0 nI$
- Magnetic field at ends of a long solenoid,
  $B = \dfrac{\mu_0 nI}{2}$

Magnetic Dipole

- Every current carrying loop is a magnetic dipole.
- Magnetic dipole moment of magnetic dipole is given by $m = NIA$.
- Direction of $\vec{m}$ is perpendicular to the plane of the loop and given by right hand screw law.

Magnetic Moment of a Revolving Electron

- $\mu_l = \dfrac{evr}{2} = \dfrac{e}{2m_e}(L)$; L = orbital angular momentum
  Vectorially, $\vec{\mu}_l = -\dfrac{e}{2m_e}\vec{L}$
- Gyromagnetic ratio, $\dfrac{\mu_l}{L} = \dfrac{e}{2m_e} = 8.8 \times 10^{10}\ \text{C kg}^{-1}$
- Bohr magneton, $\mu_B = \dfrac{eh}{4\pi m_e} = 9.27 \times 10^{-24}\ \text{A m}^2$
  It is the smallest value of magnetic moment.

Magnetic Dipole in Uniform Magnetic Field

- Net force, $\vec{F} = 0$
- Torque, $\vec{\tau} = \vec{m} \times \vec{B}$ or $\tau = mB\sin\theta$
- Potential energy, $U = -\vec{m}\cdot\vec{B} = -mB\cos\theta$
- Work done in moving a dipole from its angular position θ_1 to θ_2
  $W_{\theta_1 \to \theta_2} = U_{\theta_2} - U_{\theta_1} = mB(\cos\theta_1 - \cos\theta_2)$
- $\theta = 0°$ is stable equilibrium position of the dipole. Here, $F = 0, \tau = 0$ and $U = -mB$ = minimum.
- $\theta = 180°$ is unstable equilibrium position of the dipole. Here, $F = 0, \tau = 0$ and $U = mB$ = maximum.
- Time period of vibration of a freely suspended magnet in a uniform magnetic field
  $T = 2\pi\sqrt{\dfrac{I}{mB}}$

Magnetic Field Due to a Bar Magnet

- End-on position, $B = \dfrac{\mu_0}{4\pi}\dfrac{2mr}{(r^2 - l^2)^2}$
  For short magnet $r >> l$, $B = \dfrac{\mu_0}{4\pi}\dfrac{2m}{r^3}$
- Broadside-on position, $B = \dfrac{\mu_0}{4\pi}\dfrac{m}{(r^2 + l^2)^{3/2}}$
  For short magnet $r >> l$, $B = \dfrac{\mu_0}{4\pi}\dfrac{m}{r^3}$

Gauss's Law in Magnetism

$\oint \vec{B}\cdot d\vec{s} = 0$
- It shows magnetic monopole does not exist.
- Magnetic fields are always in closed loop.

Earth's Magnetism

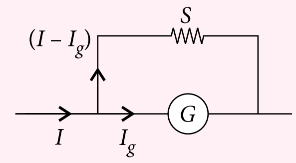
- Net magnetic field, $B = \sqrt{B_H^2 + B_V^2}$
  Here, $B_H = B\cos\delta, B_V = B\sin\delta$
  δ is the dip angle at the place.
- At equator, $B_H = B$ so, $\delta = 0°$ and $B_V = 0$.
- At poles, $B_V = B$ so, $\delta = 90°$ and $B_H = 0$
- The lines drawn through different places having same declination are called isogonic lines. A line which passes through places having zero declination is called agonic line.
- The lines which pass through different places having same dip are called isoclinic lines. A line which passes through places having zero dip is called aclinic line.
- The lines drawn through places having the same value of B_H are called isodynamic lines.

Ampere's Circuital Law

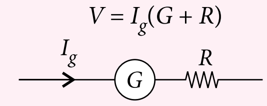
- Ampere's circuital law
  $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{en}$
- Force per unit length between two parallel current carrying wires,
  $\dfrac{F}{l} = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$
- Parallel currents attract and antiparallel currents repel.

Moving Coil Galvanometer

- Deflection, $\phi = \left(\dfrac{NAB}{\kappa}\right)I$
- Current sensitivity, $\dfrac{\phi}{I} = \dfrac{NAB}{\kappa}$
- Voltage sensitivity, $\dfrac{\phi}{V} = \left(\dfrac{NAB}{\kappa}\right)\dfrac{I}{V} = \left(\dfrac{NAB}{\kappa}\right)\dfrac{1}{R}$
- Conversion of galvanometer into ammeter
  $\dfrac{S}{G} = \dfrac{I_g}{I - I_g}$
  Shunt, $S = \dfrac{I_g G}{I - I_g}$



- An ammeter is connected in series to the wire in which current is to be found.
- Conversion of galvanometer into ammeter
  $V = I_g(G + R)$



  High resistance required in series,
  $R = \dfrac{V}{I_g} - G$
- A voltmeter is connected in parallel across the two points between which potential difference is to be found.

Magnetic Properties of Materials

- Magnetisation of material, $\vec{M} = \dfrac{\vec{m}_{net}}{V}$
- Magnetic induction due to current carrying solenoid having a core
  $B = B_0 + B_M = \mu_0 n(I + I_M)$
- Magnetic field intensity $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$
- Magnetic susceptibility, $\chi = \dfrac{M}{H}$
- Magnetic permeability, $\mu = \mu_0(1 + \chi)$
- Relative permeability $\mu_r = 1 + \chi$
- Curie law, $\chi = \dfrac{C}{T}$
- Curie-Weiss law, $\chi = \dfrac{C}{T - T_C}(T > T_C)$

Classification of Magnetic material

- Diamagnetic substances, where the total magnetic moment of all the particles is zero.
- Paramagnetic substances, where the total magnetic moment of all the particles constituting a substance is not zero and has a small value.
- Ferromagnetic substances, where the total magnetic moment of all the particles has a large value.
-

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 \le \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

COMEDK

SOLVED PAPER 2015

1. The number of electrons in one coloumb of charges is
(a) 6.25×10^{18} (b) 6.25×10^{16}
(c) 6.25×10^{10} (d) 6.25×10^{12}

2. A particle A has charge $+q$ and particle B has charge $+4q$, each of them having the same mass m. When allowed to fall from rest through the same electrical potential difference, the ratio of their speeds will become
(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

3. A stone weighs 100 N on the surface of the Earth. The ratio of its weight at a height of half the radius of the Earth to a depth of half the radius of the earth will be approximately
(a) 3.6 (b) 2.2 (c) 1.8 (d) 0.9

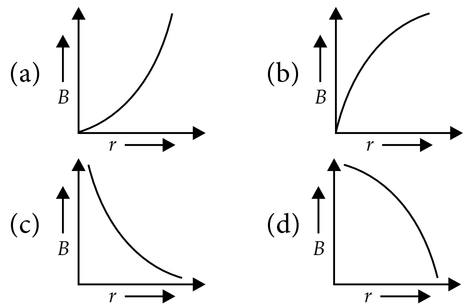
4. The gravitational field strength at the surface of a certain planet is g. Which of the following is the gravitational field strength at the surface of a planet with twice the radius and twice the mass?
(a) $g/2$ (b) g (c) $2g$ (d) $4g$

5. Vector A has a magnitude of 10 units and makes an angle of 30° with the positive x-axis. Vector B has a magnitude of 20 units and makes an angle of 30° with the negative x-axis. What is the magnitude of the resultant between these two vectors?
(a) $20\sqrt{3}$ (b) 35 (c) $15\sqrt{3}$ (d) $10\sqrt{3}$

6. A uniform chain of length L is lying partly on a table, the remaining part hanging down from the edge of the table. If the coefficient of friction between the chain and the table is 0.5, what is the minimum length of the chain that should lie on the table, to prevent the chain from slipping down to the ground?
(a) $L/3$ (b) $L/2$ (c) $2L/3$ (d) $3L/4$

7. A man weighing 70 kg, riding a motorbike weighing 230 kg at 54 km hr^{-1}, accelerates at 1 m s^{-2} for 10 s, when suddenly a child rushes into the road. The rider manages to apply brakes screeching to bring his vehicle to a halt in 3 s, just in time to save the child. What should have been the average retarding force on the vehicle?
(a) 1.5 N (b) 2.5 N (c) 3.5 N (d) 4.5 N

8. The graph showing the variation of the magnetic field strength (B) with distance (r) from a long current carrying conductor is



9. Inside a cyclotron, a charged particle is subjected to both an electric field and a magnetic field. It gains kinetic energy due to
(a) only the electric field
(b) only the magnetic field
(c) both the fields
(d) none of the fields.

10. A certain length of insulated wire can be bent to form either a single circular loop (case I) or a double loop of smaller radius (case II). When the same steady current is passed through the wire, the ratio of the magnetic field at the centre in case I to that in case II is
(a) 1 (b) 2 (c) 1/2 (d) 1/4

11. A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be

(a) a straight line
(b) a circle
(c) a helix with uniform pitch
(d) a helix with non uniform pitch

12. The correct definition of Meissner effect is
(a) The phenomenon of perfect paramagnetism in supercondutors
(b) The phenomenon of perfect diamagnetism in supercondutors
(c) The phenomenon of perfect diamagnetism in semicondutors
(d) The phenomenon of ferromagnetism in metals

13. A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of the rotation
(a) Increases
(b) Decreases
(c) Remains unchanged
(d) Doubles

14. A uniform circular disc of mass 400 g and radius 4.0 cm is rotated about one of its diameter at an angular speed of 10 rot/s. The kinetic energy of the disc is
(a) 3.2×10^{-5} J (b) 1.6×10^{-3} J
(c) 3.2×10^{-3} J (d) 6.4×10^{-4} J

15. A coil of wire of 1000 turns has self inductance 120 millihenry. The self inductance of a coil 250 turns is
(a) 60 millihenry (b) 120 millihenry
(c) 240 millihenry (d) 75 millihenry

16. The instantaneous voltage of a 50 Hz generator giving peak voltage as 300 V. The generator equation for this voltage is
(a) $V = 50 \sin 300\pi t$ (b) $V = 300 \sin 100\pi t$
(c) $V = 6 \sin 100\pi t$ (d) $V = 50 \sin 100\pi t$

17. A Jet plane is travelling west at 450 metres per second. If the horizontal component of earths magnetic field is 4×10^{-4} Tesla and angle of dip is 30°, then the vertical component is
(a) 3×10^{-4} T (b) 4×10^{-4} T
(c) 2.3×10^{-4} T (d) 12.308×10^{-4} T

18. A transformer is used to light a 100 W, 110 V lamp from a 220 V supply. If the supply current is 0.6 A, the efficiency of the transformer is
(a) 66% (b) 76% (c) 86% (d) 96%

19. A bicycle generator creates 1.5 V at 15 km/hr. The EMF generated at 10 km/hr is
(a) 1.5 volts (b) 2 volts
(c) 0.5 volts (d) 1 volts

20. The mean or average value of AC over a complete cycle is
(a) 100 (b) 0 (c) 50 (d) Infinity

21. The best waves for emission of electrons from a surface
(a) Microwaves (b) Ultra violet rays
(c) Infrared rays (d) X-rays

22. If the surface is a perfect reflector, the change in momentum of the wave after falling on the surface is
(a) P (b) $2P$ (c) $\frac{1}{2}P$ (d) $-2P$

23. In case of a *p-n* junction diode at high value of reverse bias, the current rises sharply. The value of reverse bias is known as
(a) Cut-in (b) Zener voltage
(c) Inverse voltage (d) Critical voltage

24. The truth table of a logic gate is given below

Input		Output
A	***B***	***Y***
0	0	1
0	1	1
1	0	1
1	1	0

The logic gate is
(a) OR gate (b) AND gate
(c) NOR gate (d) NAND gate

25. In a transistor connected in a common emitter mode, R_o = 4 kΩ, R_i = 1 kΩ, I_c = 1 mA and I_b = 20 mA. The voltage gain is
(a) 100 (b) 200 (c) 300 (d) 400

26. In a semiconductor diode, the reverse biased current is due to drift of the free electrons and holes caused by
(a) Thermal expansion only
(b) Impurity atoms only
(c) Both (a) and (b)
(d) Neither by (a) nor by (b)

27. An electric dipole consists of two opposite charges, each of magnitude 1.0 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. The maximum torque on the dipole is

(a) 0.2×10^{-3} N-m (b) 1×10^{-3} N-m
(c) 2×10^{-3} N-m (d) 4×10^{-3} N-m

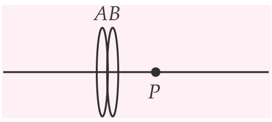
28. Two small charged spheres A and B have charges 10μC and 40μC respectively and are held at separation of 90 cm from each other. At what distance from A, electric field intensity would be zero?

(a) 22.5 cm (b) 18 cm
(c) 30 cm (d) 36 cm

29. A concave mirror gives an image three times as large as its object placed at a distance of 20 cm from it. For the image to be real, the focal length should be

(a) 10 cm (b) 15 cm
(c) 20 cm (d) 30 cm

30. Two convex lenses A and B placed in contact form the image of a distant object at P. If the lens B is moved to the right a little, the image will

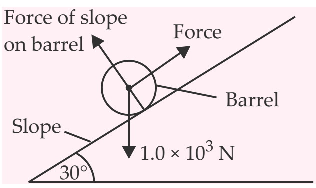


(a) Move to the left
(b) Move to the right
(c) Remain at P
(d) Move either to the left or right, depending upon focal length of the lenses

31. A equiangular glass prism of refractive index 1.6 is kept fully immersed in water of refractive index 4/3, for a certain ray of monochromatic light. What is the closest value for the angle of minimum deviation of the light ray in this setup? (Take sine 37° = 0.6)

(a) 10° (b) 14° (c) 18° (d) 22°

32. The diagram shows a barrel of weight 1.0×10^3 N on a frictionless slope inclined at 30° to the horizontal.



The force is parallel to the slope. What is the work done in moving the barrel a distance of 5.0 m up the slope?

(a) 2.5×10^3 J (b) 4.3×10^3 J
(c) 5.0×10^3 J (d) 1.0×10^3 J

33. All lights are switched off, except for a bright point-light source kept at the bottom of a swimming pool filled with clear water of refractive index 4/3. As a result, only a circular patch of 6 m diameter of the water surface is visible to spectators standing around the swimming pool. Which of the following gives the nearest value of the depth of the pool?

(a) 1.6 m (b) 2.0 m (c) 2.6 m (d) 3.0 m

34. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance X. Now the combined mass will oscillate on the spring with period

(a) $T = 2\pi\sqrt{\dfrac{mg}{X(M+m)}}$ (b) $T = 2\pi\sqrt{\dfrac{(M+m)X}{mg}}$

(c) $T = \dfrac{\pi}{2}\sqrt{\dfrac{mg}{X(M+m)}}$ (d) $T = 2\pi\sqrt{\dfrac{(M+m)}{mg}}$

35. A steel wire has a length of 90 cm which is under a constant tension of 100 N. The speed of the transverse waves that can be produced in the wire will be (take the mass of the steel wire to be 6×10^{-3} kg)

(a) 50 m/s (b) 50 cm/s
(c) $1/3\sqrt{6}$ m/s (d) $50\sqrt{6}$ m/s

36. A 10 kg collar is attached to a spring (spring constant 600 N/m), it slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 20 cm and released. What is the speed of the oscillation?

(a) $\sqrt{60} \times 0.2$ m/s (b) 60×0.2 m/s
(c) 60×2 m/s (d) 6×0.2 m/s

37. A crane with a steel cable of length 11 m and radius 2.0 cm is employed to lift a block of concrete of mass 40 tons in a building site. If the Young's Modulus of steel is 2.0×10^{11} Pa, what will be roughly the increase in the length of the cable while lifting the block? (Take $g = 10$ ms^{-2})

(a) 0.75 cm (b) 1.25 cm
(c) 1.75 cm (d) 2.50 cm

38. The dimensions of four wires of the same material are given below. In which wire the increase in length will be maximum when the same tension is applied?

(a) Length 100 cm, diameter 1 mm
(b) Length 200 cm, diameter 2 mm
(c) Length 300 cm, diameter 3 mm
(d) Length 50 cm, diameter 0.5 mm

39. Under a constant pressure head, the rate of flow of orderly volume flow of liquid through a capillary tube is V. If the length of the capillary is doubled and the diameter of the bore is halved, the rate of flow would become
(a) $V/4$ (b) $V/8$ (c) $16\ V$ (d) $V/32$

40. In an isochoric process
(a) Work done is constant
(b) Volume changes, work done remains same
(c) Volume remains constant and no work is done by the system
(d) Both volume and work done changes

41. A bomb of mass 18 kg at rest explodes into two pieces of masses 6 kg and 12 kg. The velocity of 12 kg mass is 4 m/s. The kinetic energy of the other mass is
(a) 288 J (b) 192 J (c) 96 J (d) 144 J

42. A 2 kg mass lying on a table is displaced in the horizontal direction through 50 cm. The work done by the normal reaction will be
(a) 10 J (b) 0 (c) 100 erg (d) 100 J

43. A sample of a radioactive element whose half-life is 30 s contains a million nuclei at a certain instant of time. How many nuclei will be left after 10 s?
(a) 3.33×10^5 (b) 3.78×10^5
(c) 1.11×10^5 (d) 1.26×10^5

44. A radioactive substance decays at 1/32 of its initial activity in 25 days. Its half life is
(a) 4 days (b) 25 days
(c) 5 days (d) 20 days

45. The shortest wavelengths of Paschen, Balmer and Lyman series are in the ratio
(a) 9 : 1 : 4 (b) 1 : 4 : 9
(c) 9 : 4 : 1 (d) 1 : 9 : 4

46. A nuclear fission is said to be critical when multiplication factor or K
(a) $K = 1$ (b) $K > 1$ (c) $K < 1$ (d) $K = 0$

47. The average energy of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J. The corresponding values at 600 K are
(a) 12.12×10^{-21} J (b) 8.78×10^{-21} J
(c) 6.21×10^{-21} J (d) 12.42×10^{-21} J

48. A TV tower is 120 m high. How much more height is to be added to it, if its coverage range is to become double?
(a) 120 m (b) 240 m (c) 360 m (d) 480 m

49. A message signal of frequency 10 kHz and peak value of 10 volts is used to moderate a carrier of frequency 1 MHz and peak voltage 20 volts. The modulation index and side bands produced are
(a) 0.4 and 1200 kHz, 990 kHz
(b) 0.5 and 1010 kHz, 990 kHz
(c) 0.2 and 1010 kHz, 1000 kHz
(d) 0.5 and 1500 kHz, 1000 Hz

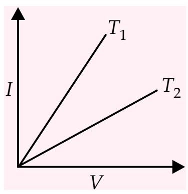
50. The wireless communication frequency bands for Cellular Mobile Radio (mobile to base station) are in range of
(a) 76-88 MHz (b) 900-1000 MHz
(c) 896-901 MHz (d) 896-1000 MHz

51. A parallel plated capacitor has area 2 m^2 separated by 3 dielectric slabs. Their relative permittivity is 2, 3, 6 and thickness is 0.4 mm, 0.6 mm, 1.2 mm respectively. The capacitance is
(a) 5×10^{-8} Farad (b) 11×10^{-8} Farad
(c) 2.95×10^{-8} Farad (d) 10×10^{-8} Farad

52. Two resistances A and B have colour codes orange, blue, white and brown, red, green respectively. Then ratio of their resistances $A : B$ is
(a) 3 : 1 (b) 1 : 3
(c) $1 : 3 \times 10^4$ (d) $3 \times 10^4 : 1$

53. Several lamps of 50 W and 100 V rating are available. How many of them can be connected in parallel across a battery of a 120 V of internal resistance 10 Ω, so that all bulbs glow in full power?
(a) 2 (b) 4 (c) 6 (d) 8

54. I and V are respectively the current and voltage in a metal wire of resistance R. The I-V graph for the two different temperatures T_1 and T_2 given, then



(a) $T_1 = T_2$ (b) $T_1 > T_2$
(c) $T_1 < T_2$ (d) $T_1 = 2T_2$

55. The momentum of electrons having a wavelength 2Å (Given $h = 6.626 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-35}$ kg) is
(a) 6.313×10^{-24} kg m s^{-1}
(b) 3.313×10^{-24} kg m s^{-1}
(c) 9.313×10^{-25} kg m s^{-1}
(d) 12.313×10^{-24} kg m s^{-1}

56. The photoelectric current of voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

(a) 24×10^{-19} J (b) -24×10^{-19} J
(c) 2.4×10^{-19} J (d) 2.04×10^{-19} J

57. A ball is dropped from the top of the building 100 m high. Simultaneously, another ball is thrown upwards from the bottom of the building with such a velocity that the balls collide exactly mid-way. What is the speed in m s^{-1} with which the second ball is thrown? (Take g = 10 m s^{-2})
(a) 31.6 (b) 27.8 (c) 22.4 (d) 19.6

58. A charge q is placed at the centre of a cube. The electric flux passing through the cube is
(a) $\frac{1}{3}\frac{q}{\varepsilon_0}$ (b) $\frac{1}{6}\frac{q}{\varepsilon_0}$ (c) $\frac{q}{\varepsilon_0}$ (d) $\frac{1}{4}\frac{q}{\varepsilon_0}$

59. A body accelerates from rest with a uniform acceleration a for a time t. The uncertainty in 'a' is 8% and the uncertainty in 't' is 4%. The uncertainty in the speed is
(a) 32% (b) 12% (c) 8% (d) 2%

60. A parallel plate capacitor has a capacity C. The separation between the plates is doubled and a dielectric medium is inserted between the plates. If the capacity is $3C$, then the dielectric constant of the medium will be
(a) 1.5 (b) 3 (c) 6 (d) 12

SOLUTIONS

1. **(a) :** $Q = ne, \; n = \frac{Q}{e}$

Here, $e = 1.6 \times 10^{-19}$ C, Q = 1 C, n = ?

$\therefore \quad n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$

2. **(b) :** Mass of each charged particle = m
Let potential difference be V.
The energy of charge $+q$ when passing through potential difference V,

$$E = qV = \frac{1}{2}mv^2$$

The energy of charge $+4q$ when passing through potential difference V,

$$E' = 4qV = \frac{1}{2}mv'^2$$

$$\therefore \quad \frac{E}{E'} = \frac{v^2}{v'^2} = \frac{qV}{4qV} = \frac{1}{4} \quad \text{or} \quad \frac{v}{v'} = \frac{1}{2}$$

3. **(d) :** Weight of a mass depends on the acceleration due to gravity (g).

Acceleration due to gravity at height $h\left(=\frac{R}{2}\right)$ from the surface of earth,

$$g_h = \frac{g}{\left(1+\frac{R/2}{R}\right)^2} = \frac{4}{9}g \quad \text{...(i)}$$

Acceleration due to gravity at depth $d\left(=\frac{R}{2}\right)$ from the surface of earth

$$g_d = g\left(1-\frac{R/2}{R}\right) = g\left(1-\frac{1}{2}\right) = \frac{g}{2} \quad \text{...(ii)}$$

Required ratio $= \frac{W_h}{W_d} = \frac{mg_h}{mg_d} = \frac{\frac{4}{9}g}{\frac{g}{2}} = \frac{8}{9} = 0.9$

4. **(a) :** For a given planet, $g = \frac{GM}{R^2}$

Gravitational field strength at the surface of another planet, $g' = \frac{GM'}{R'^2}$

Here, $M' = 2M$, $R' = 2R$ $\therefore \quad g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$

5. **(d) :** Given, $|\vec{A}|$ = 10 units, $|\vec{B}|$ = 20 units
Angle between $\vec{A}$ and $\vec{B}$ θ = 180° – 30° – 30° = 120°
Magnitude of the resultant,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$= \sqrt{10^2 + 20^2 + 2 \times 10 \times 20\cos 120^\circ}$$
$$= \sqrt{100+400-200} = \sqrt{300} = 10\sqrt{3} \text{ units}$$

6. **(c) :** Let x be length of the chain that lies on the table.
Mass per unit length of the chain $= \frac{M}{L}$
Mass of length x of the chain $= \frac{M}{L}x$
Mass of the length $(L - x)$ of hanging chain
$= \frac{M}{L}(L - x)$
At equilibrium, friction force between table and chain = weight of hanging part of chain

$$\mu\left(\frac{M}{L}x\right)g = \frac{M}{L}(L-x)g$$

$0.5\,x = L - x$; $1.5\,x = L$ $\therefore \quad x = \frac{2L}{3}$

7. **(*) :** Mass of man and motorbike
M = 70 + 230 = 300 kg
Case (i)

$$u = 54 \text{ km h}^{-1} = 54 \times \frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

t = 10 s, a = 1 m s^{-2}, v = ?
$v = u + at$ = 15 + 1 × 10 = 25 m s^{-1}

Case (ii)

Motorbike stops in 3 s to save a child.

$u = 25$ m s^{-1}, $t = 3$ s, $v = 0$, $a = ?$

Using, $v = u + at$

$$0 = 25 + a(3)\,;\, a = -\frac{25}{3}\text{ m s}^{-2}$$

Hence, retarding force on the vehicle,

$$F = M\times|a| = 300\times\frac{25}{3} = 2500\text{ N} = 2.5\text{ kN}$$

* None of the given options is correct.

8. (c) : The magnetic field strength (B) at the distance (r) from a long current carrying conductor is given by $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

For given current, Br = constant.

This represents a rectangular hyperbola.

9. (a) : Magnetic force does no work on the charged particle. Hence, gain in kinetic energy of a charged particle in a cyclotron depends only on electric field.

10. (d) : Magnetic field at the centre of a current carrying circular wire, $B = \frac{\mu_0 nI}{2r}$

Let length of the wire be L.

Case I

$$2\pi r_1 = L,\ r_1 = \frac{L}{2\pi} \qquad \therefore \quad B_1 = \frac{\mu_0 I}{2(L/2\pi)}$$

Case II

$$2\times(2\pi r_2) = L;\ r_2 = \frac{L}{4\pi} \qquad \therefore \quad B_2 = \frac{\mu_0(2I)}{2(L/4\pi)}$$

Hence, $\frac{B_1}{B_2} = \frac{1}{4}$

11. (c) **12. (b)**

13. (c) : As there is no external torque on the rotating system hence its angular momentum is conserved.

Note : Here moment of inertia about the axis of rotation decreases.

14. (*) : Here, $m = 400$ g $= 0.4$ kg, $r = 4$ cm $= 0.04$ m

$\omega = 10$ rot/s $= 10\times 2\pi$ rad s^{-1} $= 20\,\pi$ rad s^{-1}

$$\text{KE} = \frac{1}{2}I_d\,\omega^2 = \frac{1}{2}\times\left(\frac{1}{4}mr^2\right)\omega^2\left[\because\ I_d = \frac{1}{4}mr^2\right]$$

$$= \frac{1}{8}\times 0.4\times(0.04)^2\times(20\,\pi)^2 = 3.2\times 10^{-1}\text{ J}$$

* None of the given options is correct.

15. (*) : The question is ambiguous.

16. (b) : Here, frequency of generator, $\upsilon = 50$ Hz peak voltage, $V_o = 300$ V

Instantaneous voltage of the generator,

$V = V_o\sin\omega t = V_o\sin 2\pi\upsilon t = 300\sin 100\,\pi t$

17. (c) : Given, $H = 4\times10^{-4}$ T, $\delta = 30°$

As $V = H\tan\delta = 4\times10^{-4}\times\tan 30° = 2.3\times10^{-4}$ T

18. (b) : As efficiency of the transformer, $\eta = \frac{P_{out}}{P_{in}}$

Here, $P_{out} = 100$ W, $V_{in} = 220$ V

$I_{in} = 0.6$ A

$\therefore\quad P_{in} = V_{in}\times I_{in} = 220\times 0.6 = 132$ W

So, $\eta = \frac{100}{132} = 0.7576 = 75.76\% \approx 76\%$

19. (d) : Emf induced, $\varepsilon = Blv$

Here, $\vec{B}, \vec{l}$ and $\vec{v}$ are mutually perpendicular

For given B and l, $\varepsilon \propto v$. $\therefore\ \frac{\varepsilon_1}{\varepsilon_2} = \frac{v_1}{v_2}$

Here, $\varepsilon_1 = 1.5$ V, $v_1 = 15$ km/hr $= 15\times\frac{5}{18}$ m s^{-1}

$v_2 = 10$ km/hr $= 10\times\frac{5}{18}$ ms^{-1}, $\varepsilon_2 = ?$

So, $\frac{1.5}{\varepsilon_2} = \frac{15\times\frac{5}{18}}{10\times\frac{5}{18}} = \frac{3}{2};\quad \varepsilon_2 = 1$ V

20. (b) : The mean or average value of AC over a complete cycle is zero because it contains positive half cycle and negative half cycle of the physical quantity.

21. (d)

22. (d) : Let momentum of incident wave = P

Momentum of reflected wave = $-P$

(Since surface is a perfect reflector.)

Change in momentum of the wave = $-P - P = -2P$

23. (b) : In a p-n junction diode at high value of reverse bias, the current rises sharply due to zener breakdown. This high value of reverse bias is called zener voltage.

24. (d) : According to truth table, output of logic gate would be $Y = \overline{A\cdot B}$

This is output of NAND gate.

25. (*) : Here, $R_o = 4$ kΩ $= 4000\ \Omega$,

$R_i = 1$ kΩ $= 1000\ \Omega$, $I_c = 1$ mA $= 1\times10^{-3}$ A

$I_b = 20$ mA $= 20\times10^{-3}$ A, $A_v = ?$

$$A_v = \frac{V_o}{V_i} = \frac{I_cR_o}{I_bR_i} = \frac{1\times10^{-3}\times4000}{20\times10^{-3}\times1000} = 0.2$$

* None of the given options is correct.

Note : If I_b were in μA, option (b) would have been correct.

26. **(c) :** In a semiconductor diode, the reverse bias current is due to drift of the free electrons and holes which are generated by thermal expansion, impurity atoms etc.

27. **(c) :** The maximum torque on the dipole in an external electric field is given by

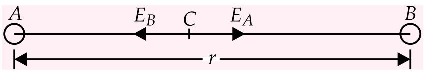
$\tau = pE = q(2a) \times E$

Here, $q = 1\ \mu C = 10^{-6}$ C, $2a = 2$ cm $= 2 \times 10^{-2}$ m, $E = 10^5$ N C^{-1}, $\tau = ?$

$\therefore\ \ \tau = 10^{-6} \times 2 \times 10^{-2} \times 10^5 = 2 \times 10^{-3}$ N m

28. **(c) :** Given situation is shown in the figure.

Suppose electric field intensity at point C is zero.



Here, $AB = r = 90$ cm $= 0.9$ m

$q_A = 10\ \mu C = 10 \times 10^{-6}$ C

$q_B = 40\ \mu C = 40 \times 10^{-6}$ C, $AC = ?$

At point C, $E_A = E_B$

$$\frac{q_A}{4\pi\varepsilon_0 (AC)^2} = \frac{q_B}{4\pi\varepsilon_0 (BC)^2}$$

$$\frac{q_A}{(AC)^2} = \frac{q_B}{(r - AC)^2} \Rightarrow \frac{10 \times 10^{-6}}{(AC)^2} = \frac{40 \times 10^{-6}}{(0.9 - AC)^2}$$

$$\frac{1}{(AC)^2} = \frac{4}{(0.9 - AC)^2};\ \frac{1}{AC} = \frac{2}{0.9 - AC}$$

$0.9 - AC = 2\,AC;\ 3\,AC = 0.9$

$AC = 0.3$ m $= 30$ cm.

29. **(b) :** Here, for real image, $m = -3$ or $-\dfrac{v}{u} = -3$

$\therefore\ \ v = 3\,u$

Also, $u = -20$ cm, so $v = -60$ cm, $f = ?$

Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\frac{1}{-60} + \frac{1}{-20} = \frac{1}{f} \Rightarrow -\frac{4}{60} = \frac{1}{f};\ \ \therefore\ \ f = -15 \text{ cm}$$

30. **(b)**

31. **(b) :** Here, angle of prism, $A = 60°$

Refractive index of prism, $\mu_P = 1.6$

Refractive index of water, $\mu_W = \dfrac{4}{3}$

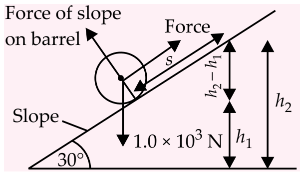
Angle of minimum deviation, $\delta_m = ?$

$$\text{As } \frac{\mu_P}{\mu_W} = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} \Rightarrow \frac{1.6}{\frac{4}{3}} = \frac{\sin\left(\dfrac{60° + \delta_m}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$$

$$1.2 = 2\sin\left(30° + \frac{\delta_m}{2}\right) \Rightarrow \sin 37° = \sin\left(30° + \frac{\delta_m}{2}\right)$$

$$37° = 30° + \frac{\delta_m}{2};\ \frac{\delta_m}{2} = 7°\ \therefore\ \ \delta_m = 14°$$

32. **(a) :** Work done in moving the barrel on the frictionless slope = change in potential energy of barrel



$W = mg\,(h_2 - h_1)$

Here, $mg = 1.0 \times 10^3$ N

$(h_2 - h_1) = s \sin 30° = 5 \sin 30° = 2.5$ m

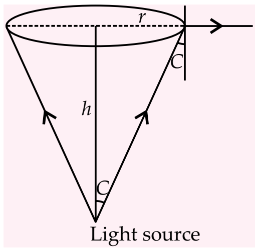
$\therefore\ \ W = 1.0 \times 10^3 \times 2.5 = 2.5 \times 10^3$ J

33. **(c) :** Given situation is shown in the figure.

$\mu_W = \dfrac{4}{3};\ r = \dfrac{D}{2} = \dfrac{6}{2} = 3$ m

$h = ?$

This situation is seen if there is phenomena of total internal reflection of light.



$$\therefore\ \ \sin C = \frac{1}{\mu_W} = \frac{3}{4}$$

Again from figure, $\sin C = \dfrac{r}{\sqrt{r^2 + h^2}};\ \dfrac{3}{4} = \dfrac{3}{\sqrt{3^2 + h^2}}$

or $16 = 9 + h^2$; $h^2 = 7$, $h = \sqrt{7}$ m ≈ 2.6 m

34. **(b) :** Time period of spring block system is given by, $T = 2\pi\sqrt{\dfrac{\text{mass of block}}{\text{spring constant}}}$

Here, mass of block $= (M + m)$

Spring constant $k = \dfrac{mg}{X}$

[$\because$ At equilibrium, $(M + m)g = k(X_o + X)$
or, $mg = kX$ (Initially, $Mg = kX_o$)]

$$\therefore\ \ T = 2\pi\sqrt{\frac{(M+m)}{\frac{mg}{X}}} = 2\pi\sqrt{\frac{(M+m)X}{mg}}$$

35. **(d) :** Speed of transverse wave, $v = \sqrt{\dfrac{T}{\mu}}$

Here, $T = 100$ N, $\ \ m = 6 \times 10^{-3}$ kg

$l = 90$ cm $= 0.9$ m

$$\mu = \frac{m}{l} = \frac{6 \times 10^{-3}}{0.9} = \frac{6}{9} \times 10^{-2} \text{ kg m}^{-1}$$

$$v = \sqrt{\frac{100}{\frac{6}{9} \times 10^{-2}}} = \sqrt{\frac{9}{6} \times 10^4} = \frac{300}{\sqrt{6}} = 50\sqrt{6}\ \text{ m s}^{-1}$$

36. (a) : Angular frequency of spring – block system is given by $\omega = \sqrt{\frac{k}{m}}$

Maximum speed in oscillation,

$v_{max} = A\omega = A\sqrt{\frac{k}{m}}$

Here, $A = 20$ cm $= 0.2$ m, $k = 600$ N m^{-1}

$m = 10$ kg, $v_{max} = ?$

$v_{max} = 0.2\sqrt{\frac{600}{10}} = 0.2\sqrt{60}$ m s^{-1}

37. (c) : Length of cable, $L = 11$ m

Radius of cable, $r = 2$ cm $= 0.02$ m

Young's modulus of steel, $Y = 2 \times 10^{11}$ Pa

$g = 10$ m s^{-2}; increase in length, $l = ?$

Mass of block, $M = 40$ tons $= 40 \times 10^3$ kg

As, $Y = \dfrac{F/A}{l/L} = \dfrac{Mg \times L}{\pi r^2 \times l}$

$$l = \frac{Mg \times L}{\pi r^2 \times Y} = \frac{40 \times 10^3 \times 10 \times 11}{3.14 \times (0.02)^2 \times 2 \times 10^{11}}$$

$$= \frac{11}{628} = 0.0175 \text{ m} = 1.75 \text{ cm.}$$

38. (d) : As $Y = \dfrac{F/A}{l/L} = \dfrac{F \times L}{\pi r^2 \times l} \Rightarrow l = \dfrac{F \times L}{\pi r^2 \times Y}$

For given F and Y, $l \propto \dfrac{L}{r^2} \propto \dfrac{L}{D^2}$

$\therefore \quad l_1 : l_2 : l_3 : l_4 = \dfrac{L_1}{D_1^2} : \dfrac{L_2}{D_2^2} : \dfrac{L_3}{D_3^2} : \dfrac{L_4}{D_4^2}$

$\dfrac{L_1}{D_1^2} = \dfrac{100\,\text{cm}}{(1\,\text{mm})^2}, \quad \dfrac{L_2}{D_2^2} = \dfrac{200\,\text{cm}}{(2\,\text{mm})^2},$

$\dfrac{L_3}{D_3^2} = \dfrac{300\,\text{cm}}{(3\,\text{mm})^2}, \quad \dfrac{L_4}{D_4^2} = \dfrac{50\,\text{cm}}{(0.5\,\text{mm})^2}$

So, $l_1 : l_2 : l_3 : l_4 = 1 : \dfrac{1}{2} : \dfrac{1}{3} : 2$

Clearly l_4 is maximum. So correct option is (d).

39. (d) : According to Poiseuille's formula, rate of flow through a narrow tube $V = \dfrac{\pi P r^4}{8\eta l}$

For given P and η, $V \propto \dfrac{r^4}{l} \quad \therefore \quad \dfrac{V_1}{V_2} = \dfrac{r_1^4}{r_2^4} \times \dfrac{l_2}{l_1}$

Here, $V_1 = V$, $r_2 = r_1/2$, $l_2 = 2l_1$, $V_2 = ?$

So, $\dfrac{V}{V_2} = (2)^4 \times 2 = 32, \quad V_2 = \dfrac{V}{32}$

40. (c) : In an isochoric process, volume of system remains constant, so work done ($W = P\Delta V$) by the system is zero.

41. (b) : Using momentum conservation principle,

$M \times 0 = m_1v_1 + m_2v_2; \quad m_2v_2 = -m_1v_1 \qquad \ldots$(i)

Kinetic energy of m_2,

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}\frac{(m_2v_2)^2}{m_2}$$

$$= \frac{1}{2}\frac{(-m_1v_1)^2}{m_2} = \frac{(m_1v_1)^2}{2m_2} \qquad \text{[Using eqn. (i)]}$$

Here, $m_1 = 12$ kg, $m_2 = 6$ kg, $v_1 = 4$ m s^{-1}, $K_2 = ?$

$\therefore \quad K_2 = \dfrac{(12 \times 4)^2}{2 \times 6} = 192$ J

42. (b) : Normal reaction on the mass by the table is perpendicular to the horizontal displacement. Hence work done by normal reaction will be zero.

43. (*) : As $N = N_o\left(\frac{1}{2}\right)^{t/T_{1/2}}$

Here, $N_o = 10^6$, $t = 10$ s, $T_{1/2} = 30$ s

$\therefore \quad N = 10^6\left(\frac{1}{2}\right)^{10/30} = 10^6 \times 0.794 = 7.94 \times 10^5$

* None of the given options is correct.

44. (c) : Here, $t = 25$ days, $R = \left(\frac{1}{32}\right)R_o$, $\tau_{1/2} = ?$

As $R = R_o e^{-\lambda t}$ or $\dfrac{R}{R_o} = e^{-\left(\frac{\ln 2}{\tau_{1/2}} \times t\right)} \quad \left[\because \tau_{1/2} = \dfrac{\ln 2}{\lambda}\right]$

$32 = 2^{t/\tau_{1/2}}; \; 2^5 = 2^{t/\tau_{1/2}} \; \therefore \; \tau_{1/2} = \dfrac{t}{5} = \dfrac{25}{5} = 5$ days.

45. (c) : The shortest wavelength of Paschen (λ_P) Balmer (λ_B) and Lyman (λ_L) series are given by

$\dfrac{1}{\lambda_P} = \dfrac{R_H}{3^2}; \; \dfrac{1}{\lambda_B} = \dfrac{R_H}{2^2}$ and $\dfrac{1}{\lambda_L} = \dfrac{R_H}{1^2}$

So, $\lambda_P = \dfrac{9}{R_H}, \quad \lambda_B = \dfrac{4}{R_H}, \quad \lambda_L = \dfrac{1}{R_H}$

$\therefore \quad \lambda_P : \lambda_B : \lambda_L = 9 : 4 : 1$

46. (a)

47. (d) : The average kinetic energy of molecules in a sample of oxygen gas is given by

$KE_{avg} = \dfrac{3}{2}kT \quad$ or $\quad \dfrac{(KE_{avg})_1}{(KE_{avg})_2} = \dfrac{T_1}{T_2}$

Here, $(KE_{avg})_1 = 6.21 \times 10^{-21}$ J, $T_1 = 300$ K

$T_2 = 600$ K, $(KE_{avg})_2 = ?$

$\therefore \quad \dfrac{6.21\times10^{-21}}{(KE_{avg})_2} = \dfrac{300}{600} = \dfrac{1}{2}$

$(KE_{avg})_2 = 12.42 \times 10^{-21}$ J

48. **(c) :** Coverage range of TV tower is given by $d = \sqrt{2Rh}$

As R is constant, so $\dfrac{d_1}{d_2} = \sqrt{\dfrac{h_1}{h_2}}$

Here, $d_2 = 2d_1$, $h_1 = 120$ m, $(h_2 - h_1) = ?$

$\dfrac{d_1}{2d_1} = \sqrt{\dfrac{120}{h_2}}$; $h_2 = 480$ m

Height to be added to the tower $= h_2 - h_1$
$= 480 - 120 = 360$ m

49. **(b) :** Modulation index, $\mu = \dfrac{A_m}{A_c}$

Side band frequencies are
USB $= \upsilon_c + \upsilon_m$, LSB $= \upsilon_c - \upsilon_m$
Here, $A_m = 10$ V, $A_c = 20$ V
$\upsilon_c = 1$ MHz $= 1000$ kHz, $\upsilon_m = 10$ kHz

$\therefore \quad \mu = \dfrac{10}{20} = 0.5$

USB = 1000 + 10 = 1010 kHz
LSB = 1000 – 10 = 990 kHz

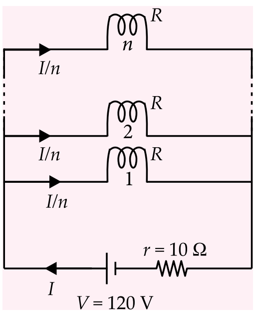
50. **(c) :** The wireless communication frequency bands for cellular mobile radio are in the range of 896-901 MHz (mobile to base station) and 840-935 MHz (base station to mobile).

51. **(c)**

52. **(d) :** Resistance $A = 36 \times 10^9\ \Omega$
Resistance $B = 12 \times 10^5\ \Omega$

$\therefore \quad \dfrac{A}{B} = \dfrac{36\times10^9}{12\times10^5} = 3\times10^4 \Rightarrow A : B = 3 \times 10^4 : 1$

53. **(b) :** Given situation is shown in the figure
Suppose there are n bulbs.
Resistance of each bulb,



$R = \dfrac{V^2}{P} = \dfrac{100^2}{50} = 200\ \Omega$

Using KVL in lower loop,

$V - Ir - \dfrac{I}{n}R = 0;$

$120 - 10I - \dfrac{I}{n}\times 200 = 0$

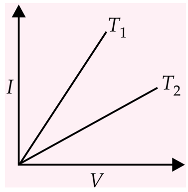
$12 - I = \dfrac{20I}{n}$ ...(i)

Power delivered by battery = Sum of power used in bulbs $(120 - 10\,I)I = 50\,n$; $(12 - I)I = 5\,n$

$\left(\dfrac{20I}{n}\right)I = 5n; \dfrac{I^2}{n^2} = \dfrac{1}{4}; \therefore \quad I = \dfrac{n}{2}$ [using eqn. (i)]

From eqn. (i) $12 - \dfrac{n}{2} = \dfrac{10n}{n}; \dfrac{n}{2} = 12 - 10 \ \therefore \ n = 4$

54. **(c) :** Slope of line $= \dfrac{1}{R}$



or $\quad R = \dfrac{1}{\text{slope}}$

$\because \quad (\text{slope})_{T_2} < (\text{slope})_{T_1}$

$\therefore \quad R_2 > R_1$

Resistance of a metal wire increases with the temperature. So $T_2 > T_1$.

55. **(b) :** Here, $\lambda = 2$ Å $= 2 \times 10^{-10}$ m,
$h = 6.626 \times 10^{-34}$ J s, $m = 9.1 \times 10^{-35}$ kg, $p = ?$

As, $\lambda = \dfrac{h}{p}; p = \dfrac{h}{\lambda}$

$p = \dfrac{6.626\times10^{-34}}{2\times10^{-10}} = 3.313\times10^{-24}$ kg m s^{-1}

56. **(c) :** Maximum kinetic energy of a photoelectron,
$K_{max} = eV_0 = 1.6 \times 10^{-19} \times 1.5 [\because V_0 = 1.5 \text{ V}]$
$= 2.4 \times 10^{-19}$ J

Note : Here, 1.5 V is the stopping potential of a photoelectron.

57. **(a)**

58. **(c) :** Electric flux, $\phi = \dfrac{q_{en}}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$

59. **(b) :** Using kinematic eqn, $v = u + at = at\ [\because u = 0]$

For error calculation, $\dfrac{\Delta v}{v} = \dfrac{\Delta a}{a} = \dfrac{\Delta t}{t}$

Here, $\dfrac{\Delta a}{a} = 8\%$, $\dfrac{\Delta t}{t} = 4\%$

So, $\dfrac{\Delta v}{v} = 8 + 4 = 12\%$

60. **(c) :** Capacity of a parallel plate capacitor,

$C = \dfrac{\varepsilon_0 A}{d}$ ...(i)

When plate separation is increased and a dielectric is inserted, its capacity becomes $3C$.

$\therefore \quad 3C = \dfrac{\varepsilon_0 KA}{d'} = \dfrac{\varepsilon_0 KA}{2d} \quad (\because d' = 2d)$

$3C = \dfrac{K}{2}C$ [Using eqn. (i)]

Hence, $K = 6$

KCET

SOLVED PAPER 2015

1. A proton beam enters a magnetic field of 10^{-4} Wb m^{-2} normally. If the specific charge of the proton is 10^{11} C kg^{-1} and its velocity is 10^{9} m s^{-1}, then the radius of the circle described will be

(a) 10 m (b) 1 m
(c) 0.1 m (d) 100 m

2. Two concentric coils each of radius equal to 2π cm are placed right angles to each other. If 3 A and 4 A are the currents flowing through the two coils respectively. The magnetic induction (in Wb m^{-2}) at the centre of the coils will be

(a) 10^{-5} (b) 7×10^{-5}
(c) 12×10^{-5} (d) 5×10^{-5}

3. The resistance of the bulb filament is 100 Ω at a temperature of 100°C. If its temperature co-efficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature

(a) 400°C (b) 200°C
(c) 300°C (d) 500°C

4. In Wheatstone's network $P = 2\ \Omega$, $Q = 2\ \Omega$, $R = 2\ \Omega$ and $S = 3\ \Omega$. The resistance with which S is to shunted in order that the bridge may be balanced is

(a) 2 Ω (b) 6 Ω
(c) 1 Ω (d) 4 Ω

5. Core of electromagnets are made of ferromagnetic material which has

(a) high permeability and high retentivity
(b) low permeability and low retentivity
(c) high permeability and low retentivity
(d) low permeability and high retentivity

6. If there is no torsion in the suspension thread, then the time period of a magnet executing SHM is

(a) $T = \frac{1}{2\pi}\sqrt{\frac{I}{MB}}$ (b) $T = 2\pi\sqrt{\frac{MB}{I}}$
(c) $T = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$ (d) $T = 2\pi\sqrt{\frac{I}{MB}}$

7. Two parallel wires 1 m apart carry currents of 1 A and 3 A respectively in opposite directions. The force per unit length acting between these two wires is

(a) 6×10^{-7} N m^{-1} attractive
(b) 6×10^{-5} N m^{-1} attractive
(c) 6×10^{-7} N m^{-1} repulsive
(d) 6×10^{-5} N m^{-1} repulsive

8. A galvanometer of resistance 50 Ω gives a full scale deflection for a current 5×10^{-4} A. The resistance that should be connected in series with the galvanometer to read 3 V is

(a) 5050 Ω (b) 5950 Ω
(c) 595 Ω (d) 5059 Ω

9. A cyclotron is used to accelerate

(a) only positively charged particles
(b) both positively and negatively charged particles
(c) neutron
(d) only negatively charged particles

10. A transformer is used to light 100 W–110 V lamp from 220 V mains. If the main current is 0.5 A, the efficiency of the transformer is

(a) 95% (b) 99% (c) 90% (d) 96%

11. In an LCR circuit, at resonance

(a) the impedance is maximum
(b) the current leads the voltage by $\pi/2$
(c) the current and voltage are in phase
(d) the current is minimum

12. An aircraft with a wingspan of 40 m flies with a speed of 1080 km/hr in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of the earth's magnetic field 1.75×10^{-5} T. Then the emf developed between the tips of the wings is
(a) 0.34 V (b) 2.1 V (c) 0.5 V (d) 0.21 V

13. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to the equation $i = i_m \sin\omega t$ where $i_m = 10$ A and $\omega = 100\pi$ rad s^{-1}. The maximum value of the emf induced in the second coil is
(a) 5π (b) 4π (c) 2π (d) π

14. The magnetic susceptibility of a paramagnetic material at –73°C is 0.0075 and its value at –173°C will be
(a) 0.0030 (b) 0.0075
(c) 0.0045 (d) 0.015

15. In a Young's double slit experiment the slit separation is 0.5 m from the slits. For a monochromatic light of wavelength 500 nm, the distance of 3rd maxima from 2nd minima on the other side is
(a) 2.5 mm (b) 2.25 mm
(c) 2.75 mm (d) 22.5 mm

16. Calculate the focal length of a reading glass of a person if his distance of distinct vision is 75 cm.
(a) 37.5 cm (b) 100.4 cm
(c) 25.6 cm (d) 75.2 cm

17. A person wants a real image of his own, 3 times enlarged. Where should he stand infront of a concave mirror of radius of curvature 30 cm?
(a) 30 cm (b) 20 cm
(c) 10 cm (d) 90 cm

18. If ε_0 and μ_0 are the permittivity and permeability of free space and ε and μ are the corresponding quantities for a medium, then refractive index of the medium is
(a) $\sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}}$ (b) insufficient information
(c) $\sqrt{\dfrac{\mu_0\varepsilon_0}{\mu\varepsilon}}$ (d) 1

19. The average power dissipated in a pure inductor is
(a) VI^2 (b) zero
(c) $\frac{1}{2}VI$ (d) $\frac{VI^2}{4}$

20. An α-particle of energy 5 MeV is scattered through 180° by gold nucleus. The distance of closest approach is of the order of
(a) 10^{-12} cm (b) 10^{-16} cm
(c) 10^{-10} cm (d) 10^{-14} cm

21. Find the de-Broglie wavelength of an electron with kinetic energy of 120 eV.
(a) 102 pm (b) 124 pm (c) 95 pm (d) 112 pm

22. Light of two different frequencies whose photons have energies 1 eV and 2.5 eV respectively, successively illuminate a metallic surface whose work function is 0.5 eV. Ratio of maximum speeds of emitted electrons will be
(a) 1 : 4 (b) 1 : 1 (c) 1 : 5 (d) 1 : 2

23. The polarizing angle of glass is 57°. A ray of light which is incident at this angle will have an angle of refraction as
(a) 33° (b) 38° (c) 25° (d) 43°

24. To observe diffraction, the size of the obstacle
(a) should be $\lambda/2$, where λ is the wavelength.
(b) should be of the order of wavelength.
(c) has no relation to wavelength.
(d) should be much larger than the wavelength.

25. A radioactive decay can form an isotope of the original nucleus with the emission of particles
(a) one α and two β (b) four α and one β
(c) one α and four β (d) one α and one β

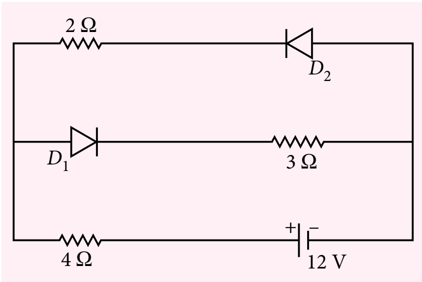
26. The half life of a radioactive substance is 20 minutes. The time taken between 50% decay and 87.5% decay of the substance will be
(a) 40 minutes (b) 10 minutes
(c) 30 minutes (d) 25 minutes

27. A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio
(a) 6 : 1 (b) 2 : 1 (c) 8 : 1 (d) 4 : 1

28. What is the wavelength of light for the least energetic photon emitted in the Lyman series of the hydrogen spectrum? (Take hc = 1240 eV nm)
(a) 102 nm (b) 150 nm
(c) 82 nm (d) 122 nm

29. If an electron in hydrogen atom jumps from an orbit of level n = 3 to an orbit of level n = 2, the emitted radiation has a frequency
(R = Rydberg constant, C = velocity of light)
(a) $\frac{RC}{25}$ (b) $\frac{5RC}{36}$ (c) $\frac{3RC}{27}$ (d) $\frac{8RC}{9}$

30. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit?



(a) 2.0 A (b) 1.33 A (c) 1.71 A (d) 2.31 A

31. Amplitude modulation has
(a) one carrier
(b) one carrier with high frequency
(c) one carrier with two side band frequencies
(d) one carrier with infinite frequencies

32. An LED is constructed from a *pn* junction based on a certain semi-conducting material whose energy gap is 1.9 eV. Then the wavelength of the emitted light is
(a) 1.6×10^{-8} m (b) 9.1×10^{-5} m
(c) 2.9×10^{-9} m (d) 6.5×10^{-7} m

33. The waves used for line-of-sight (LOS) communication is
(a) space waves (b) sky waves
(c) ground waves (d) sound waves

34. The given truth table is for

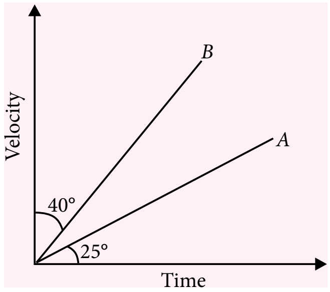
Input		Output
A	*B*	*Y*
0	0	1
0	1	1
1	0	1
1	1	0

(a) OR gate (b) NOR gate
(c) AND gate (d) NAND gate

35. The input characteristics of a transistor in *CE* mode is the graph obtained by plotting
(a) I_B against V_{CE} at constant V_{BE}
(b) I_B against I_C at constant V_{BE}
(c) I_B against V_{BE} at constant V_{CE}
(d) I_B against I_C at constant V_{CE}

36. A particle is projected with a velocity v so that its horizontal range twice the greatest height attained. The horizontal range is
(a) $\frac{2v^2}{3g}$ (b) $\frac{v^2}{2g}$ (c) $\frac{v^2}{g}$ (d) $\frac{4v^2}{5g}$

37. The velocity – time graph for two bodies *A* and *B* are shown. Then the acceleration of *A* and *B* are in the ratio



(a) tan25° to tan50° (b) cos25° to cos50°
(c) tan25° to tan40° (d) sin25° to sin50°

38. The ratio of the dimensions of Planck constant and that of moment of inertia has the dimensions of
(a) frequency (b) velocity
(c) time (d) angular momentum

39. Moment of inertia of a thin uniform rod rotating about the perpendicular axis passing through its centre is *I*. If the same rod is bent into a ring and its moment of inertia about its diameter is I', then the ratio $\frac{I}{I'}$ is
(a) $\frac{8}{3}\pi^2$ (b) $\frac{5}{3}\pi^2$ (c) $\frac{3}{2}\pi^2$ (d) $\frac{2}{3}\pi^2$

40. If the mass of a body is *M* on the surface of the earth, the mass of the same body on the surface of the moon is
(a) *M* (b) zero (c) $\frac{M}{6}$ (d) 6*M*

41. The ratio of angular speed of a second-hand to the hour-hand of a watch is
(a) 60 : 1 (b) 72 : 1 (c) 720 : 1 (d) 3600 : 1

42. The kinetic energy of a body of mass 4 kg and momentum 6 N s will be
(a) 3.5 J (b) 5.5 J (c) 2.5 J (d) 4.5 J

43. A stone of mass 0.05 kg is thrown vertically upwards. What is the direction and magnitude of net force on the stone during its upward motion?
(a) 0.49 N vertically downwards
(b) 9.8 N vertically downwards
(c) 0.49 N vertically upwards
(d) 0.98 N vertically downwards

44. The ratio of kinetic energy to the potential energy of a particle executing SHM at a distance equal to half its amplitude, the distance being measured from its equilibrium position is
(a) 4 : 1 (b) 8 : 1 (c) 3 : 1 (d) 2 : 1

45. 1 gram of ice is mixed with 1 gram of steam. At thermal equilibrium, the temperature of the mixture is
(a) 100°C (b) 55°C (c) 0°C (d) 50°C

46. Water is heated from 0°C to 10°C, then its volume
(a) increases
(b) first decreases and then increases
(c) decreases
(d) does not change

47. The efficiency of a Carnot engine which operates between the two temperatures T_1 = 500 K and T_2 = 300 K is
(a) 25% (b) 40% (c) 50% (d) 75%

48. The ratio of hydraulic stress to the corresponding strain is known as
(a) Bulk modulus (b) Rigidity modulus
(c) Compressibility (d) Young's modulus

49. The angle between the dipole moment and electric field at any point on the equatorial plane is
(a) 90° (b) 45° (c) 0° (d) 180°

50. Pick out the statement which is incorrect.
(a) The electric field lines forms closed loop.
(b) Field lines never intersect.
(c) The tangent drawn to a line of force represents the direction of electric field.
(d) A negative test charge experiences a force opposite to the direction of the field.

51. Two spheres carrying charges +6 μC and +9 μC, separated by a distance *d*, experiences a force of repulsion *F*. When a charge of –3 μC is given to both the sphere and kept at the same distance as before, the new force of repulsion is
(a) 3*F* (b) $\frac{F}{9}$ (c) *F* (d) $\frac{F}{3}$

52. A stretched string is vibrating in the second overtone, then the number of nodes and antinodes between the ends of the string are respectively
(a) 3 and 2 (b) 2 and 3
(c) 4 and 3 (d) 3 and 4

53. When two tuning forks *A* and *B* are sounded together, 4 beats per second are heard. The frequency of the fork *B* is 384 Hz. When one of the prongs of the fork *A* is filed and sounded with *B*, the beat frequency increases, then the frequency of the fork *A* is
(a) 388 Hz (b) 389 Hz
(c) 380 Hz (d) 379 Hz

54. Three resistances 2 Ω, 3 Ω and 4 Ω are connected in parallel. The ratio of currents passing through them when a potential difference is applied across its ends will be
(a) 6 : 4 : 3 (b) 4 : 3 : 2
(c) 6 : 3 : 2 (d) 5 : 4 : 3

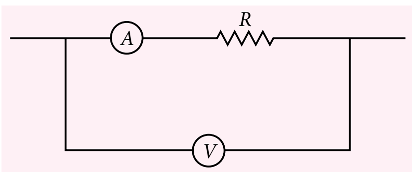
55. Four identical cells of emf *E* and internal resistance *r* are to be connected in series. Suppose if one of the cell is connected wrongly, the equivalent emf and effective internal resistance of the combination is
(a) 4*E* and 2*r* (b) 2*E* and 2*r*
(c) 4*E* and 4*r* (d) 2*E* and 4*r*

56. A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential and capacitance respectively are
(a) increases, decreases, decreases
(b) constant, increases, decreases
(c) constant, decreases, decreases
(d) constant, decreases, increases

57. A spherical shell of radius 10 cm is carrying a charge *q*. If the electric potential at distances 5 cm, 10 cm and 15 cm from the centre of the spherical shell is V_1, V_2 and V_3 respectively, then
(a) $V_1 < V_2 < V_3$ (b) $V_1 = V_2 < V_3$
(c) $V_1 > V_2 > V_3$ (d) $V_1 = V_2 > V_3$

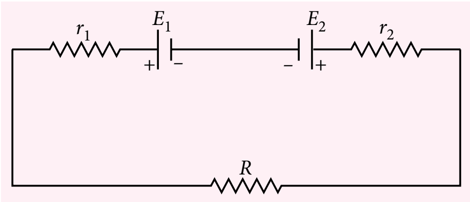
58. Three point charges 3 nC, 6 nC and 9 nC are placed at the corners of an equilateral triangle of side 0.1 m. The potential energy of the system is
(a) 89100 J (b) 99100 J
(c) 8910 J (d) 9910 J

59. In the circuit shown below, the ammeter and the voltmeter readings are 3 A and 6 V respectively. Then the value of the resistance *R* is



(a) > 2 Ω (b) ≥ 2 Ω (c) 2 Ω (d) < 2 Ω

60. Two cells of emf E_1 and E_2 are joined in opposition (such that $E_1 > E_2$). If r_1 and r_2 be the internal resistance and *R* be the external resistance, then the terminal potential difference is



(a) $\dfrac{E_1+E_2}{r_1+r_2+R}\times R$ (b) $\dfrac{E_1-E_2}{r_1+r_2+R}\times R$

(c) $\dfrac{E_1+E_2}{r_1+r_2}\times R$ (d) $\dfrac{E_1-E_2}{r_1+r_2}\times R$

SOLUTIONS

1. **(d):** When the proton beam enters the magnetic field B normally, it describes a circular path of radius r given by

$$r=\frac{mv}{eB}=\frac{v}{\frac{e}{m}B}$$

where $\frac{e}{m}$ is the specific charge of the proton and v is its velocity.

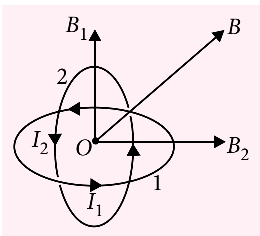
Here,

$$v=10^9\ \text{m s}^{-1},\ \frac{e}{m}=10^{11}\ \text{C kg}^{-1}$$

$$B=10^{-4}\ \text{Wb m}^{-2}$$

$$\therefore\quad r=\frac{10^9\ \text{m s}^{-1}}{(10^{11}\ \text{C kg}^{-1})(10^{-4}\ \text{Wb m}^{-2})}=100\ \text{m}$$

2. **(d):** The situation is shown in the figure.



For coil 1,

Radius, $R_1=2\pi$ cm $=2\pi\times10^{-2}$ m

Current, $I_1=3$ A

For coil 2,

Radius, $R_2=2\pi$ cm $=2\pi\times10^{-2}$ m

Current, $I_2=4$ A

The magnetic induction at the centre O due to the current I_1 in coil 1 is

$$B_1=\frac{\mu_0 I_1}{2R_1}$$

It acts in vertically upwards direction.

The magnetic induction at the centre O due to the current I_2 in coil 2 is

$$B_2=\frac{\mu_0 I_2}{2R_2}$$

It acts in the horizontal direction.

As B_1 and B_2 are perpendicular to each other, so the net magnetic field at the centre O is

$$B=\sqrt{B_1^2+B_2^2}$$

$$=\sqrt{\left(\frac{\mu_0 I_1}{2R_1}\right)^2+\left(\frac{\mu_0 I_2}{2R_2}\right)^2}$$

Substituting the given values, we get

$$B=\sqrt{\left(\frac{\mu_0(3\ \text{A})}{2(2\pi\times10^{-2}\ \text{m})}\right)^2+\left(\frac{\mu_0(5\ \text{A})}{2(2\pi\times10^{-2}\ \text{m})}\right)^2}$$

$$=\frac{\mu_0\sqrt{(3\ \text{A})^2+(4\ \text{A})^2}}{2(2\pi\times10^{-2}\ \text{m})}$$

$$=\frac{(4\pi\times10^{-7}\ \text{Wb m}^{-1}\ \text{A}^{-1})(5\ \text{A})}{(4\pi\times10^{-2}\ \text{m})}$$

$$=5\times10^{-5}\ \text{Wb m}^{-2}$$

3. **(a):** As the resistance of the bulb filament at T°C is

$$R_T=R_0(1+\alpha T)$$

where R_0 is its resistance at 0°C and α is the temperature coefficient of resistance.

$\therefore\quad R_{T_1}=R_0(1+\alpha T_1)$ ...(i)

and $\quad R_{T_2}=R_0(1+\alpha T_2)$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{R_{T_1}}{R_{T_2}}=\frac{1+\alpha T_1}{1+\alpha T_2}$$

Here,

$R_{T_1}=100\ \Omega,\ T_1=100$°C

$R_{T_2}=200\ \Omega,\ T_2=?$

$\alpha=0.005$°C^{-1}

$$\therefore\quad \frac{100\ \Omega}{200\ \Omega}=\frac{1+(0.005°\text{C}^{-1})(100°\text{C})}{1+(0.005°\text{C}^{-1})T_2}$$

$$\frac{1}{2}=\frac{1+0.5}{1+(0.005°\text{C}^{-1})T_2}$$

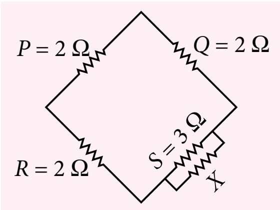
$$2(1+0.5)=1+(0.005°\text{C}^{-1})T_2$$

$$3=1+(0.005°\text{C}^{-1})T_2$$

or $\quad(0.005°\text{C}^{-1})T_2=3-1=2$

$$T_2=\frac{2}{(0.005°\text{C}^{-1})}=400°\text{C}$$

4. (b): Let X be the resistance with which S is to be shunted for the bridge to be balanced. Then, for balanced Wheatstone's bridge



$$\frac{P}{Q}=\frac{R}{\frac{SX}{S+X}}$$ (∵ S and X are in parallel)

Substituting the given values, we get

$$\frac{2\ \Omega}{2\ \Omega}=\frac{2\ \Omega}{\frac{(3\ \Omega)X}{3\ \Omega+X}} \quad \text{or} \quad \frac{(3\ \Omega)\,X}{3\ \Omega+X}=2\ \Omega$$

$$(3\ \Omega)X = (2\ \Omega)(3\ \Omega) + (2\ \Omega)X$$
$$X(3\ \Omega - 2\ \Omega) = (2\ \Omega)(3\ \Omega)$$
$$X(1\ \Omega) = (2\ \Omega)(3\Omega)$$
$$X=\frac{(2\ \Omega)(3\ \Omega)}{1\ \Omega}=6\ \Omega$$

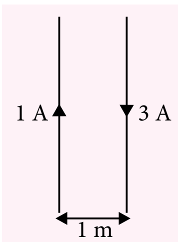
5. (c): Core of electromagnets are made of ferromagnetic material which has high permeability and low retentivity.

6. (d): The time period of a magnet executing SHM is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where I is the moment of inertia of the magnet, M is its magnetic moment and B is an external magnetic field.

7. (c): The force per unit length acting between two parallel wires carrying currents I_1 and I_2 and placed a distance d apart is



$$f=\frac{\mu_0 I_1 I_2}{2\pi d}$$

Here, $I_1 = 1$ A, $I_2 = 3$ A, $d = 1$ m

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

$$\therefore \quad f=\frac{(4\pi\times10^{-7}\ \text{T m A}^{-1})(1\ \text{A})(3\ \text{A})}{2\pi(1\ \text{m})}$$

$= 6 \times 10^{-7}$ N m^{-1}

As the currents are in opposite directions, so f is repulsive.

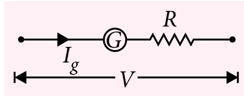
8. (b): Here,

Resistance of galvanometer, $G = 50\ \Omega$

Current for full scale deflection, $I_g = 5 \times 10^{-4}$ A

Let R be resistance connected in series with the galvanometer to read 3 V. Then

$$V = I_g(G + R)$$



or $$R = \frac{V}{I_g} - G$$

$$R = \frac{3\ \text{V}}{5\times10^{-4}\ \text{A}} - 50\ \Omega$$

$= 6000\ \Omega - 50\ \Omega = 5950\ \Omega$

9. (b): A cyclotron is used to accelerate both positively and negatively charged particles.

10. (c): The efficiency of the transformer is

$$\eta = \frac{\text{Output power}\,(P_{out})}{\text{Input power}(P_{in})} \times 100$$

Here,

$P_{out} = 100$ W

$P_{in} = (220\ \text{V})(0.5\ \text{A}) = 110$ W

$$\therefore \quad \eta = \frac{100\ \text{W}}{110\ \text{W}} \times 100 \approx 90\%$$

11. (c): In an LCR circuit, the phase difference(ϕ) between current and voltage is

$$\tan\phi = \frac{X_C - X_L}{R}$$

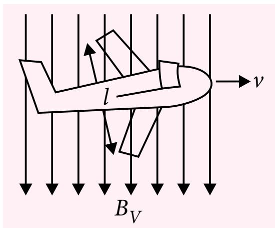
At resonance, $X_C = X_L$

∴ $\tan\phi = 0$ or $\phi = \tan^{-1}(0) = 0°$

Thus the current and voltage are in phase.

The current is maximum and the impedance is minimum at resonance in an LCR circuit.

12. (d):



The emf developed between the tips of the wings is

$$\varepsilon = B_V l v$$

Here,

$B_V = 1.75 \times 10^{-5}$ T, $l = 40$ m

$v = 1080$ km/hr $= 1080 \times \frac{5}{18}$ m s^{-1}

$= 300$ m s^{-1}

∴ $\varepsilon = (1.75 \times 10^{-5}\ \text{T})(40\ \text{m})(300\ \text{m s}^{-1})$

$= 0.21$ V

13. (a) : The emf induced in the second coil is

$$\varepsilon = M\frac{di}{dt}$$

where M is the mutual inductance of two coils, $\frac{di}{dt}$ is the rate of change of current in the first coil.

But $\frac{di}{dt} = \frac{d}{dt}(i_m \sin\omega t) = i_m\omega\cos\omega t$

$\therefore \quad \varepsilon = Mi_m\omega\cos\omega t$

For maximum value of emf induced, $\cos\omega t = 1$

$\therefore$ The maximum value of the emf induced is

$$\varepsilon_{max} = Mi_m\omega$$

Here,

$$M = 0.005 \text{ H}, i_m = 10 \text{ A}, \omega = 100\pi \text{ rad s}^{-1}$$

$\therefore \quad \varepsilon_{max} = (0.005 \text{ H})(10 \text{ A})(100\pi) = 5\pi \text{ V}$

14. (d): According to Curie's law, the magnetic susceptibility (χ) of a paramagnetic substance is inversely proportional to absolute temperature T

i.e., $\chi \propto \frac{1}{T} \quad \therefore \quad \frac{\chi_1}{\chi_2} = \frac{T_2}{T_1} \quad$ or $\quad \chi_2 = \chi_1\frac{T_1}{T_2}$

Here,

$\chi_1 = 0.0075$

$T_1 = -73°\text{C} = (273 - 73) \text{ K} = 200 \text{ K}$

$T_2 = -173°\text{C} = (273 - 173) \text{ K} = 100 \text{ K}$

$\therefore \quad \chi_2 = (0.0075)\left(\frac{200 \text{ K}}{100 \text{ K}}\right) = 0.015$

15. Data is insufficient.

16. (a) : The distance of normal vision is 25 cm. So if a book to be read is at $u = -25$ cm, its image should be formed at $v = -75$ cm. Therefore, the required focal length (f) is

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-75 \text{ cm}} - \frac{1}{-25 \text{ cm}}$$

$$= \frac{-1}{75 \text{ cm}} + \frac{1}{25 \text{ cm}} = \frac{-1+3}{75 \text{ cm}} = \frac{2}{75 \text{ cm}}$$

$$f = \frac{75 \text{ cm}}{2} = 37.5 \text{ cm}$$

17. (b): As radius of curvature (R) is negative for a concave mirror, so

$$R = -30 \text{ cm}$$

$\therefore$ The focal length of the mirror is

$$f = \frac{R}{2} = \frac{-30 \text{ cm}}{2} = -15 \text{ cm}$$

As the person wants his real enlarged image, so magnification, $m = -\frac{v}{u} = -3 \quad$ or $\quad v = 3u$

According to mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \therefore \quad \frac{1}{3u} + \frac{1}{u} = \frac{1}{-15 \text{ cm}}$$

$$\frac{1+3}{3u} = \frac{1}{-15 \text{ cm}} \quad \text{or} \quad \frac{4}{3u} = \frac{1}{-15 \text{ cm}}$$

$$u = \frac{-60 \text{ cm}}{3} = -20 \text{ cm}$$

–ve sign shows that the person should stand infront of the mirror.

18. (a) : Refractive index of the medium is

$$n = \frac{c(\text{speed of light in vacuum})}{v(\text{speed of light in medium})}$$

But $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \quad n = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{1}{\sqrt{\mu\varepsilon}}} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

19. (b): The average power dissipated is

$P_{av} = VI\cos\phi$ where $\cos\phi$ is the power factor.

For pure inductor, $\phi = \frac{\pi}{2}$

$\therefore \quad \cos\phi = 0 \quad$ or $\quad P_{av} = 0$

Thus the average power dissipated in a pure inductor is zero.

20. (a) : At the distance of closest approach d,

Kinetic energy of α-particle

= Potential energy of α-particle and gold nucleus

i.e., $K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{d} = \frac{2Ze^2}{4\pi\varepsilon_0 d} \quad$ or $\quad d = \frac{2Ze^2}{4\pi\varepsilon_0 K}$

Here,

$K = 5 \text{ MeV} = 5 \times 1.6 \times 10^{-13} \text{ J}$

($\because$ 1 MeV $= 1.6 \times 10^{-13}$ J)

For gold, $Z = 79$

$$\therefore \quad d = \frac{(2)(9\times10^9 \text{ N m}^2 \text{ C}^{-2})(79)(1.6\times10^{-19} \text{ C})^2}{(5\times1.6\times10^{-13} \text{ J})}$$

$= 4.55 \times 10^{-14}$ m $\approx 10^{-12}$ cm

21. (d): The de-Broglie wavelength of an electron is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

where h is the Planck's constant, m is the mass of the electron and K is its kinetic energy.

Here,

$h = 6.63 \times 10^{-34}$ J s

$m = 9.1 \times 10^{-31}$ kg

$K = 120$ eV $= 120 \times 1.6 \times 10^{-19}$ J

$(\because\ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$

$$\therefore \quad \lambda = \frac{6.63\times10^{-34}\text{ J s}}{\sqrt{2(9.1\times10^{-31}\text{ kg})(120\times1.6\times10^{-19}\text{ J})}}$$

$= 1.12 \times 10^{-10}$ m $= 112 \times 10^{-12}$ m

$= 112$ pm $\quad (\because\ 1 \text{ pm} = 10^{-12} \text{ m})$

22. (d): According to Einstein's photoelectric equation, the maximum kinetic energy of emitted photoelectrons is

$$K_{max} = h\upsilon - \phi_0$$

where $h\upsilon$ is the energy of incident photon and ϕ_0 is the work function.

But $K_{max} = \frac{1}{2}mv_{max}^2 \quad \therefore \quad \frac{1}{2}mv_{max}^2 = h\upsilon - \phi_0$

As per question

$$\frac{1}{2}mv_{max_1}^2 = 1 \text{ eV} - 0.5 \text{ eV} = 0.5 \text{ eV} \qquad \text{...(i)}$$

and $$\frac{1}{2}mv_{max_2}^2 = 2.5 \text{ eV} - 0.5 \text{ eV} = 2 \text{ eV} \qquad \text{...(ii)}$$

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{v_{max_1}^2}{v_{max_2}^2} = \frac{0.5 \text{ eV}}{2 \text{ eV}} = \frac{1}{4}, \quad \frac{v_{max_1}}{v_{max_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

23. (a): As the ray of light is incident at glass at polarizing angle (i_p), so the reflected and refracted rays are perpendicular to each other.

If r is the angle of refraction, then $i_p + r = 90°$

$r = 90° - i_p$

Here, $i_p = 57° \therefore r = 90° - 57° = 33°$

24. (b): To observe diffraction, the size of the obstacle should be the order of wavelength.

25. (a): As the emission of an alpha particle decreases the atomic number and mass number by 2 and 4 respectively while the emission of a beta particle increases the atomic number by 1 but mass number remains the same. So, the emission of one alpha particle and two beta particles from the nucleus can form an isotope of the original nucleus.

It may be represented as

$${}_ZX^A \xrightarrow{\alpha} {}_{Z-2}Y^{A-4} \xrightarrow{2\beta^-} {}_ZX^{A-4}$$

26. (a)

27. (c) : Let A_1 and A_2 be the mass numbers of the two nuclear parts. Their radii are given by

$$R_1 = R_0A_1^{1/3} \qquad \text{...(i)}$$

and $$R_2 = R_0A_2^{1/3} \qquad \text{...(ii)}$$

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \quad \text{or} \quad \frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^3$$

As $\frac{R_1}{R_2} = \frac{1}{2}$ (given) $\therefore \frac{A_1}{A_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Hence the ratio of their masses is

$$\frac{m_1}{m_2} = \frac{1}{8} \qquad \text{...(iii)}$$

According to law of conservation of linear momentum

magnitude of p_1 = magnitude of p_2

i.e., $m_1v_1 = m_2v_2$

or $$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{8}{1} \qquad \text{(using (iii))}$$

28. (d)

29. (b): When an electron jumps from higher level n_1 to lower energy level n_2, the frequency of the emitted radiation is

$$\upsilon = RC\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$$

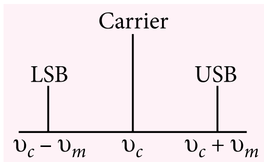
$\therefore$ For $n = 3$ to $n = 2$,

$$\upsilon = RC\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = RC\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$= RC\left[\frac{9-4}{36}\right] = \frac{5RC}{36}$$

30. (c)

31. (c) : Amplitude modulation has one carrier with two side band frequencies called upper side band and lower side band. The frequency spectrum of an amplitude modulated wave is shown in the adjacent figure.



32. (d): The wavelength of the emitted light is

$$\lambda = \frac{hc}{E_g}$$

where h is the Planck's constant, c is the velocity of light in vacuum and E_g is the energy gap.

Here,

$E_g = 1.9 \text{ eV} = 1.9 \times 1.6 \times 10^{-19}\text{ J}$

$h = 6.63 \times 10^{-34}\text{ J s}, c = 3 \times 10^8 \text{ m s}^{-1}$

$$\therefore \quad \lambda = \frac{(6.63\times10^{-34}\text{J s})(3\times10^{8}\text{ m s}^{-1})}{(1.9\times1.6\times10^{-19}\text{ J})}$$

$= 6.5 \times 10^{-7}\text{m}$

33. (a) : Space waves are used for line-of-sight (LOS) communication as well as satellite communication.

34. (d) : The given truth table is for NAND gate.

35. (c) : The input characteristics of a transistor in *CE* mode is the graph obtained by plotting base current I_B against base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE}.

36. (c)

37. (a) : As the slope of velocity-time graph gives acceleration, so acceleration $a = \tan\theta$

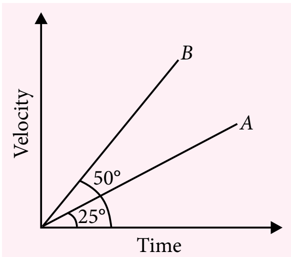
$\therefore$ From given graph,

The acceleration of A is

$a_A = \tan 25°$

and that of B is

$a_B = \tan 50°$



Their corresponding ratio is $\dfrac{a_A}{a_B} = \dfrac{\tan 25°}{\tan 50°}$

38. (a) **39. (d)**

40. (a) : The mass of the body on the surface of the moon is *M* because it doesn't depend on the acceleration due to gravity and remains the same at all places.

41. (c)

42. (d): The kinetic energy *K* and momentum *p* of a body are related as

$K = \dfrac{p^2}{2m}$ where m is the mass of the body.

Here, $p = 6$ N s, $m = 4$ kg

$$\therefore \quad K = \frac{(6\text{ N s})^2}{2(4\text{ kg})} = 4.5\text{ J}$$

43. (a) : Here,

Mass of the stone, $m = 0.05$ kg

The magnitude of net force on the stone is

$F = mg = (0.05\text{ kg})(9.8\text{ m s}^{-2}) = 0.49\text{ N}$

Its direction is vertically downwards.

44. (c)

45. (a)

46. (b): Water is heated from 0°C to 10°C, then its volume first decreases and then increases.

47. (b): The efficiency of a Carnot engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

Here, $T_1 = 500$ K, $T_2 = 300$ K

$$\therefore \quad \eta = 1 - \frac{300\text{ K}}{500\text{ K}} = \frac{500\text{ K} - 300\text{ K}}{500\text{ K}}$$

$$= \frac{200\text{ K}}{500\text{ K}} = \frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$$

48. (a) : The ratio of hydraulic stress to the corresponding strain is known as bulk modulus.

49. (d): The direction of electric field ($\vec{E}$) at any point on the equatorial line is opposite to the dipole moment ($\vec{p}$). Thus the angle between the $\vec{p}$ and $\vec{E}$ is 180°.

50. (a) : The electric field lines do not form closed loop. All other statements are correct.

51. (d) **52. (c)** **53. (a)** **54. (a)**

55. (d) **56. (b)** **57. (d)**

58. (*) : The potential energy of the system of three charges is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

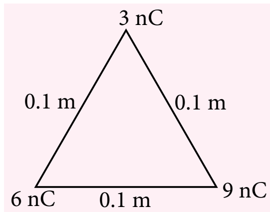
Here,

$q_1 = 3\text{ nC} = 3 \times 10^{-9}\text{ C}$

$q_2 = 6\text{ nC} = 6 \times 10^{-9}\text{ C}$

$q_3 = 9\text{ nC} = 9 \times 10^{-9}\text{ C}$

$r_{12} = r_{13} = r_{23} = 0.1\text{ m}$



$$\therefore \quad U = (9\times10^9\text{ N m}^2\text{ C}^{-2})\left[\frac{(3\times10^{-9}\text{C})(6\times10^{-9}\text{C})}{0.1} + \frac{(3\times10^{-9}\text{C})(9\times10^{-9}\text{C})}{0.1\text{ m}} + \frac{(6\times10^{-9}\text{C})(9\times10^{-9}\text{C})}{0.1\text{ m}}\right]$$

$$U = \frac{(9\times10^{9}\text{ N m}^2\text{ C}^{-2})}{(0.1\text{ m})}[(18\times10^{-18}\text{C}^2) + (27\times10^{-18}\text{C}^2) + (54\times10^{-18}\text{ C}^2)]$$

$$= \frac{(9\times10^{9}\text{ N m}^2\text{ C}^{-2})(99\times10^{-18}\text{C}^2)}{(0.1\text{ m})}$$

$= 8910 \times 10^{-9}$ J

* None of the given options is correct.

59. (d) **60. (b)**

Refer to ***MTG Karnataka CET Explorer*** *for complete solutions.*

ACE YOUR WAY CBSE XII

Series 2

CHAPTERWISE PRACTICE PAPER : Current Electricity | Magnetic Effect of Current and Magnetism

Time Allowed : 3 hours

Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory. There are 26 questions in all.

(ii) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.

(iii) Section A contains five questions of one mark each, Section B contains five questions of two marks each, Section C contains twelve questions of three marks each, Section D contains one value based question of four marks and Section E contains three questions of five marks each.

(iv) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all the three questions of five marks weightage. You have to attempt only one of the choices in such questions.

SECTION-A

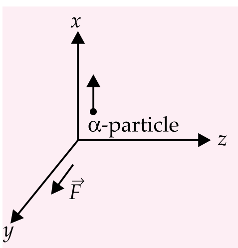
1. Name the three materials whose resistivity decreases with rise of temperature.

2. A certain electric motor wires carry a current of 5 A perpendicular to magnetic field of 0.8 T. What is the force on each cm of the wires?

3. A steady current is flowing in a cylindrical conductor. Does electric field exist within the conductor?

4. Which physical quantity has the unit Wb m^{-2}? Is it a scalar or a vector quantity?

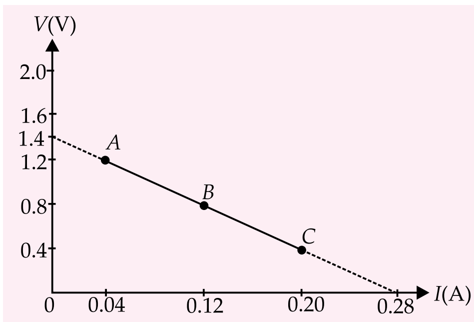
5. A beam of α particles projected along positive x-axis, experience a force due to a magnetic field along the positive y-axis as shown in figure. What is the direction of magnetic field?



SECTION-B

6. The number density of free electrons in a copper conductor is estimated as 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3 m long to its other end? The area of cross-section of the wire is 2×10^{-6} m^2 and it is carrying a current of 3 A.

7. Potential difference across terminals of a cell were measured (in V) against different currents (in A) flowing through the cell. A graph was drawn which was a straight line ABC. Using the data given in the graph determine, (i) the emf and (ii) the internal resistance of the cell.



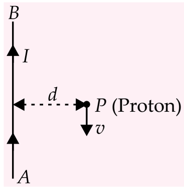
8. A proton and an α-particle enter a uniform magnetic field perpendicularly, with the same speed. How many times is the time period of the α-particle than that of the proton? Deduce an expression for the ratio of the radii of the circular paths of two particles.

OR

A charge Q is spread uniformly over an insulated ring of radius R. What is the magnetic moment of the ring if it is rotated with an angular velocity ω with respect to normal axis?

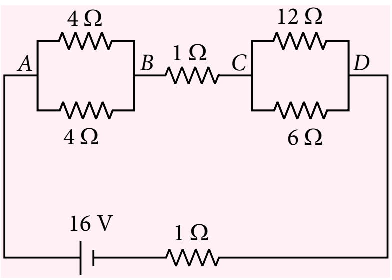
9. Write the relation for the force $\vec{F}$ acting on a charge q moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ in vector notation. Using this relation, deduce the conditions under which this force will be (i) maximum (ii) minimum.

10. A long straight wire AB carries a current I. A proton P travels with a speed v, parallel to the wire, at a distance d from it in a direction opposite to the current as shown in figure. What is the force experienced by the proton and what is its direction?

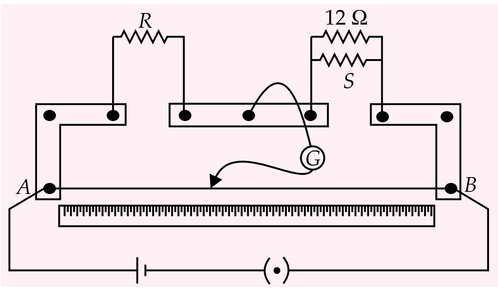


SECTION-C

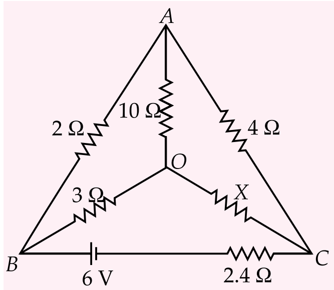
11. A network of resistances is connected to a 16 V battery with internal resistance of 1 Ω as shown in figure.
(a) Compute the equivalent resistance of the network.
(b) Obtain the voltage drop V_{AB}, V_{BC} and V_{CD}.



12. In a meter bridge, the null point is found at a distance of 40 cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50.0 cm from A. Determine the values of R and S.



13. Find the value of the unknown resistance X, in the following circuit, if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf 6 V and negligible internal resistance.

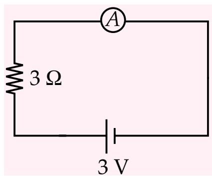


14. An n number of identical cells, each of emf ε and internal resistance r connected in series are charged by a dc source of emf ε', using a resistor R.
(i) Draw the circuit arrangement.
(ii) Deduce the expression for
(a) the charging current
(b) the potential difference across the combination of the cells.

15. A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2 A. If the number of turns is 1000 per metre, calculate (i) H (ii) B (iii) intensity of magnetisation I, and (iv) the magnetising current.

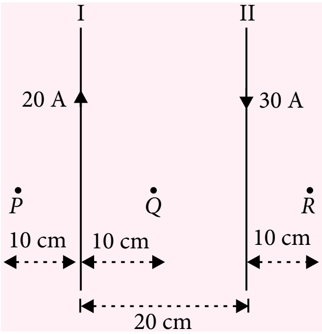
16. A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of moment 1.5×10^{-23} J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K?

17. In the circuit shown in figure, the current is to be measured. What is the value of the current if the ammeter shown
(i) is a galvanometer with a resistance G = 60 Ω. (ii) is a galvanometer described in (i) but converted to an ammeter by a shunt resistance S = 0.02 Ω;
(iii) is an ideal ammeter with zero resistance?



18. Where on the earth's surface is the value of vertical component of the earth's magnetic field zero? The horizontal component of the earth's magnetic field at a given place is 0.4×10^{-4} Wb m^{-2} and angle of dip is 30°. Calculate the value of (i) vertical component, (ii) the total intensity of the earth's magnetic field.

19. In the figure two long parallel current carrying wires I and II are shown. Find the magnitudes and directions of the magnetic field induction at the points P, Q and R in the plane of paper.



20. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its dees is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator.
[$e = 1.60 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, 1 MeV $= 1.6 \times 10^{-13}$ J].

21. Define the term potential gradient. With the help of a circuit diagram, explain how a potentiometer can be used to compare the emfs of two primary cells.

22. A domain in ferromagnetic iron is in the form of a cube of side length 10^{-4} m. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g mole^{-1} and its density is 7.9 g cm^{-3}. Assume that each iron atom has a dipole moment of 9.27×10^{-24} A m^2.

OR

A proton (charge 1.6×10^{-19} C, mass $m = 1.67 \times 10^{-27}$ kg) is shot with a speed 8×10^6 m s^{-1} at angle of 30° with the x-axis. A uniform magnetic field B = 0.30 T exists along the x-axis. Show that the path of the proton is helix and find the radius of the helix.

SECTION-D

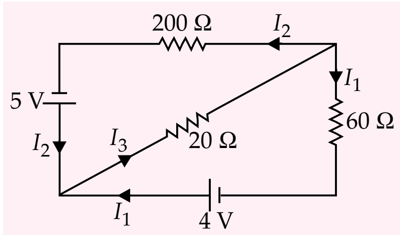
23. It is desired to supply a current of 2 A through a resistance of 10 Ω. As many as 20 cells are provided, each of emf 2 V and internal resistance 0.5 Ω. Two friends Sanju and Shyam try their hand on the problem. Sanju succeeds but Shyam fails.
Answer the following questions on the basis of given passage.
(i) Justify the set up of Sanju?
(ii) What might have gone wrong with Shyam when he gets 1.4 A current in the external load?
(iii) What are the basic values shown by Sanju and Shyam in their work?

SECTION-E

24. Three cells are connected in parallel with their like poles connected together with wires of negligible resistance. If the emfs of the cells are 2 V, 1 V and 4 V respectively and their internal resistances are 4 Ω, 3 Ω and 2 Ω respectively, find the current through each cell.

OR

State Kirchhoff's rules. Use these rules to determine the value of currents I_1, I_2 and I_3 in the circuit as shown.



25. State Biot-Savart's law. Using this law derive an expression for the magnetic field at a point situated at a distance of x metre from the centre of a circular coil of N turns and radius r carrying a current of I A.

OR

Explain the difference between diamagnetic, paramagnetic and ferromagnetic substances.

26. Explain the principle and working of a cyclotron with the help of a neat diagram. Write the expression for cyclotron frequency.

OR

A long straight wire of uniform cross section of radius a is carrying a steady current I. Use Ampere's circuital law to obtain a relation showing the variation of the magnetic field (B) inside and outside the wire with distance r, ($r < a$) and ($r > a$) at the field point from the centre of its cross section. Plot a graph showing the nature of this variation.

SOLUTIONS

1. Silicon, germanium and carbon.
2. Here, I = 5 A, B = 0.8 T, l = 1 cm = 0.01 m
Thus, $F = IlB = (5 \times 0.01 \times 0.8)$ N = 0.04 N
3. Yes, electric field exists within the conductor because it is the electric field which imparts acceleration to electrons for the flow of current.
4. Wb m^{-2} is the SI unit of magnetic field intensity B. It is a vector quantity.
5. According to Fleming's Left Hand rule, the direction of magnetic field is along negative z-axis.

6. Here, $n = 8.5 \times 10^{28}\ m^{-3}$, $l = 3$ m
$A = 2 \times 10^{-6}\ m^2$, $I = 3$ A

As, $I = n\,Ae\,v_d$ $\therefore\ v_d = \dfrac{I}{nAe}$,

Now, $t = \dfrac{l}{v_d} = \dfrac{ln\,Ae}{I}$

$$= \frac{3 \times 8.5 \times 10^{28} \times 2 \times 10^{-6} \times 1.6 \times 10^{-19}}{3}$$

$= 2.72 \times 10^4$ s

7. (i) When no current is drawn from a cell, potential difference across terminals of the cell is equal to its emf.
From the graph it is clear that, emf = 1.4 V

(ii) As $V = \varepsilon - Ir$ or $r = \dfrac{\varepsilon - V}{I}$, $\varepsilon = 1.4$ V
Consider any given value of potential difference from graph, say

$$V = 1.2 \text{ V}$$

Current corresponding to this potential difference is $I = 0.04$ A

Thus, $r = \dfrac{1.4 - 1.2}{0.04} = 5\ \Omega$

8. Time period of revolution of a charged particle in a magnetic field,

For proton, $T_1 = \dfrac{2\pi m}{Bq}$

For α-particle, $T_2 = \dfrac{2\pi(4m)}{B(2q)} = 2 \times \dfrac{2\pi m}{Bq} = 2T_1$

The time period of α-particle is twice as that of proton.

As $r = \dfrac{mv}{Bq}$, for a given values of v and B, $r \propto \dfrac{m}{q}$

$$\therefore\ \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

OR

Charge on the element of length dl of the ring is,

$$dq = \frac{Q}{2\pi R} dl$$

Current due to circular motion of this charge is,

$$dI = dq \times \upsilon = \frac{Q}{2\pi R} dl \times \frac{\omega}{2\pi} \quad (\text{As, } \omega = 2\pi\upsilon)$$

Magnetic moment due to current dI,

$$dM = dI \times \pi R^2 = \frac{Q}{2\pi R} dl \times \frac{\omega}{2\pi} \times \pi R^2$$

or $$M = \frac{Q\omega R}{4\pi} \int dl = \frac{Q\omega R}{4\pi} 2\pi R \quad (\because \int dl = 2\pi R)$$

$$= \frac{1}{2} Q\omega R^2$$

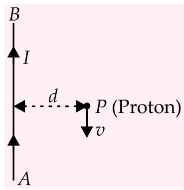
9. Force, $\vec{F} = q(\vec{v} \times \vec{B})$ or $|\vec{F}| = q\,|\vec{v} \times \vec{B}| = qvB\sin\theta$

(i) F will be maximum, when $\sin\theta = 1$ or $\theta = 90°$, *i.e.*, the charged particle is moving perpendicular to the direction of magnetic field.

(ii) F will be minimum, when $\sin\theta = 0$ or $\theta = 0°$ or $180°$ *i.e.*, the charged particle is moving parallel or anti-parallel to the direction of magnetic field.

10. Magnetic field induction at P due to current I in long straight wire AB is $B = \dfrac{\mu_0}{4\pi}\dfrac{2I}{d}$

It acts perpendicular to the plane of paper inwards.



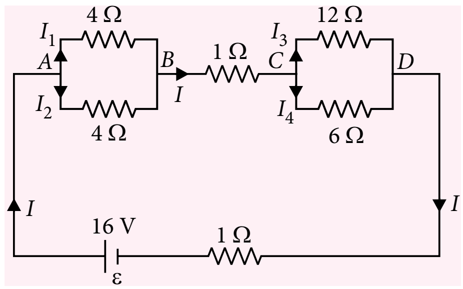
Since the proton is moving in opposite direction to the current carrying straight wire, hence the proton is moving perpendicular to the direction of magnetic field due to current through straight wire.

$\therefore$ The force acting on the proton is

$$F = qvB\sin 90° = ev\frac{\mu_0 2I}{4\pi d} = \frac{\mu_0 Iev}{2\pi d}$$

It acts in the plane of paper away from the wire.

11. The current in the various branches as shown in the figure.



(a) In the given circuit 4 Ω and 4 Ω are in parallel, 12 Ω and 6 Ω are in parallel. These two combination of resistances are in series of the circuit.

$\therefore$ Total resistance of circuit between A and D is,

$R = R_{AB} + R_{BC} + R_{CD}$

$$= \frac{4 \times 4}{4+4} + 1 + \frac{12 \times 6}{12+6} = 2 + 1 + 4 = 7\ \Omega$$

(b) Total current, $I = \dfrac{\varepsilon}{(R + r)} = \dfrac{16}{7+1} = 2$ A

Current at A is divided equally in each of 4 Ω resistances in parallel.

So, $I_1 = I_2 = 1$ A.

Potential difference across C and D,

$V_{CD} = I \times R_{CD} = 2 \times 4 = 8$ V

$$I_3 = \frac{V_{CD}}{12} = \frac{8}{12} = \frac{2}{3}\text{ A} ;\ I_4 = \frac{V_{CD}}{6} = \frac{8}{6} = \frac{4}{3}\text{ A}$$

$V_{AB} = I \times R_{AB} = 2 \times 2 = 4$ V

$V_{BC} = I \times R_{BC} = 2 \times 1 = 2$ V

$V_{CD} = I \times R_{CD} = 2 \times 4 = 8$ V

12. When resistances R and S are connected then balance point is found at a distance 40 cm from the zero end.

$$\therefore \frac{R}{S} = \frac{40}{100-40}$$

$$\frac{R}{S} = \frac{40}{60} \text{ or } \frac{R}{S} = \frac{2}{3} \qquad \text{... (i)}$$

When a resistance of 12 Ω is connected in parallel with S then total resistance in the right gap is

$$S_1 = \frac{12S}{S+12} \qquad \text{...(ii)}$$

Since balance point is obtained at a distance of 50 cm from the zero end.

$$\therefore \frac{R}{S_1} = \frac{50}{100-50} \text{ or } \frac{R}{S_1} = \frac{50}{50}$$

$$\therefore \frac{R}{S_1} = 1 \qquad \text{...(iii)}$$

Dividing (i) by (iii), we get

$$\therefore \frac{\frac{R}{S}}{\frac{R}{S_1}} = \frac{\frac{2}{3}}{1} \text{ or } \frac{R}{S} \times \frac{S_1}{R} = \frac{2}{3}$$

$$\text{or } \frac{S_1}{S} = \frac{2}{3} \text{ or } S_1 = \frac{2}{3}S$$

Putting the value of S_1 in (ii), we get

$$\frac{2}{3}S = \frac{12S}{S+12} \text{ or } \frac{2}{3} = \frac{12}{S+12}$$

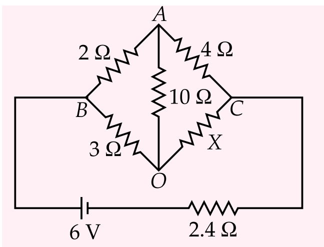
or $2S + 24 = 36$ or $2S = 12$ $\therefore S = 6\ \Omega$

Putting the value of S in (i), we get

$$\frac{R}{6} = \frac{2}{3} \Rightarrow R = \frac{2}{3} \times 6 = 4\ \Omega$$

$\therefore$ $R = 4\ \Omega$ and $S = 6\ \Omega$

13. Circuit can be rearranged as shown in the figure.



As no current flows through the section AO, this is balanced Wheatstone bridge.

$$\text{As } \frac{P}{Q} = \frac{R}{S},\ \frac{2}{4} = \frac{3}{X}$$

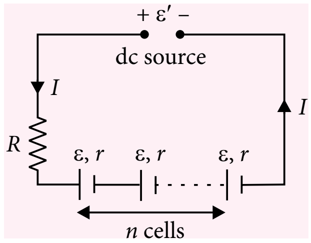
or $X = 6\ \Omega$

Equivalent resistance of the network is

$$= \frac{9 \times 6}{9+6} + 2.4 = 3.6 + 2.4 = 6\ \Omega$$

Current in the circuit is $I = \frac{V}{R} = \frac{6}{6} = 1$ A

14. (i)



(ii) (a) Net emf of cells = $n\varepsilon$

Net internal resistance = nr

So, charging current in the circuit is

$$I = \frac{\varepsilon' - n\varepsilon}{R - nr}$$

(b) Potential difference across the combination of cells is $V = n\varepsilon + Inr$

$$\text{or } V = n\varepsilon + \left(\frac{\varepsilon' - n\varepsilon}{R + nr}\right)nr$$

$$= \frac{n\varepsilon(R+nr) + (\varepsilon' - n\varepsilon)nr}{R+nr}$$

$$= \frac{n\varepsilon R + n^2\varepsilon r + n\varepsilon' r - n^2\varepsilon r}{R+nr} = \frac{n(\varepsilon R + \varepsilon' r)}{R+nr}$$

15. Here, $\mu_r = 400$, $I' = 2$ A, $n = 1000$ per metre

(i) $H = n\,I' = 1000 \times 2 = 2 \times 10^3$ A m^{-1}

(ii) $B = \mu H = \mu_0\mu_r H = 4\,\pi \times 10^{-7} \times 400 \times (2 \times 10^3)$
$= 1.0$ T

(iii) $B = \mu_0(H + I)$, where I is intensity of magnetisation,

$$I = \frac{B}{\mu_0} - H = \frac{1.0}{4\pi \times 10^{-7}} - 2 \times 10^3$$

$= 7.96 \times 10^5 - 0.02 \times 10^5 = 7.94 \times 10^5$ A m^{-1}.

(iv) The magnetising current I_m is the additional current that needs to be passed through the windings of the solenoid in the absence of the core, which would produce a B value as in the presence of the core.

Thus, $B = \mu_0 n(I' + I_m)$

$$1.0 = 4\pi \times 10^{-7} \times 1000\,(2 + I_m)$$

$$I_m = \frac{1.0}{4\pi \times 10^{-4}} - 2 = 796 - 2 = 794 \text{ A}$$

16. Here, number of dipoles, $n = 2 \times 10^{24}$
Magnetic moment of each dipole,

$$M' = 1.5 \times 10^{-23} \text{ J T}^{-1}$$

Total dipole moment of sample $= n \times M'$

$$= 2 \times 10^{24} \times 1.5 \times 10^{-23} = 30 \text{ J T}^{-1}$$

As saturation achieved is 15%, therefore, effective dipole moment,

$$M_1 = \frac{15}{100} \times 30 = 4.5 \text{ J T}^{-1},$$

$B_1 = 0.64$ T, $T_1 = 4.2$ K, $B_2 = 0.98$ T, $T_2 = 2.8$ K
According to Curie's law,

$$\chi_m = \frac{C}{T} = \frac{I}{H} \quad \text{or} \quad I = \frac{CH}{T}$$

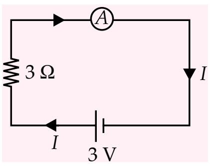
As $I \propto M$ and $H \propto B$ $\quad\therefore\quad M \propto \frac{B}{T}$,

$$\frac{M_2}{M_1} = \frac{B_2}{B_1} \cdot \frac{T_1}{T_2}$$

or $\quad M_2 = \frac{B_2 T_1 M_1}{T_2 B_1} = \frac{0.98 \times 4.2 \times 4.5}{2.8 \times 0.64}$

$$M_2 = 10.34 \text{ J T}^{-1}$$

17. (i) Let G be the resistance of ammeter (*i.e.* galvanometer). Then current in the circuit,



$$I = \frac{V}{R + G} = \frac{3}{3 + 60} = 0.048 \text{ A}$$

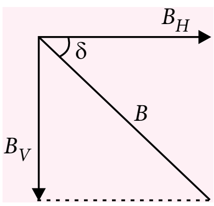
(ii) When the ammeter (*i.e.*, galvanometer) is shunted with resistance S, its effective resistance, $R_P = \frac{GS}{G + S} = \frac{60 \times 0.02}{60 + 0.02} \approx 0.02\ \Omega$

Current in the circuit,

$$I = \frac{V}{R + R_P} = \frac{3}{3 + 0.02} = 0.99 \text{ A}$$

(iii) For the ideal ammeter with zero resistance, current, $I = \frac{3}{3} = 1$ A

18. Vertical component of earth's magnetic field is zero at the equator.
$B_H = 0.4 \times 10^{-4}$ Wb m^{-2},
$\delta = 30°$



(i) $B_V = B \sin\delta$
$B_H = B \cos\delta$

$$B = \frac{B_H}{\cos\delta}$$

$$B_V = \frac{B_H \sin\delta}{\cos\delta} = B_H \tan\delta$$

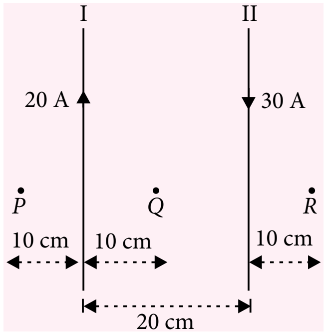
$$B_V = 0.4 \times 10^{-4} \times \tan 30°$$

$$= \frac{0.4 \times 10^{-4}}{\sqrt{3}} = 0.23 \times 10^{-4} \text{ Wb m}^{-2}$$

(ii) $B = \frac{B_H}{\cos\delta} = \frac{0.4 \times 10^{-4}}{\cos 30°}$

$B = 0.46 \times 10^{-4}$ Wb m^{-2}

19.



Resultant magnetic field induction at P is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2I_2}{r_2}$$

$$= \frac{\mu_0}{4\pi} \times 2\left[\frac{I_1}{r_1} - \frac{I_2}{r_2}\right]$$

Here, $I_1 = 20$ A, $r_1 = 0.1$ m;
$I_2 = 30$ A ; $r_2 = 0.30$ m

$$B = 10^{-7} \times 2\left[\frac{20}{0.1} - \frac{30}{0.3}\right] = 2 \times 10^{-5} \text{ T}$$

It will be acting perpendicular to the plane of paper upwards.
Resultant magnetic field induction at Q is

$$B = \frac{\mu_0}{4\pi} \times 2\left[\frac{20}{0.1} + \frac{30}{0.1}\right] = 10^{-7} \times 2 \times 500 = 10^{-4} \text{ T}$$

It will be acting perpendicular to the plane of the paper downwards.
Resultant magnetic field induction at R is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2I_2}{r_2} = 2 \times 10^{-5} \text{ T}$$

It will be acting perpendicular to the plane of the paper upwards.

20. For the controlled motion of charged particle the oscillator frequency and the frequency of accelerating proton must match.
So, frequency of proton is 10 MHz.

$$\upsilon = \frac{Bq}{2\pi m}$$

Required magnetic field $B = \dfrac{2\pi m \upsilon}{q}$

$$B = \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 10 \times 10^{6}}{1.6 \times 10^{-19}}$$

$B = 0.66$ T

Maximum velocity is attained by the proton beam at surface of dee

$$v_{max} = \frac{BqR}{m}$$

Kinetic energy, $E = \dfrac{1}{2} m v_{max}^2 = \dfrac{B^2 q^2 R^2}{2m}$

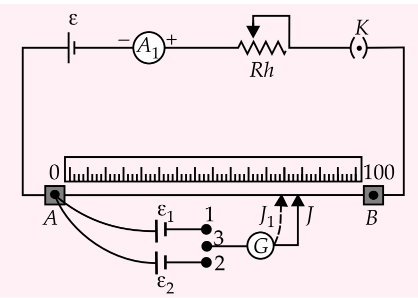
$$E = \frac{(0.66)^2 \times \left(1.6 \times 10^{-19}\right)^2 \times \left(60 \times 10^{-2}\right)^2}{2 \times 1.67 \times 10^{-27}}$$

$E = 0.12 \times 10^{-11}$ Joule

or $E = \dfrac{0.12 \times 10^{-11}}{1.6 \times 10^{-13}}$ MeV = 7.5 MeV

21. Fall of potential per unit length across a wire carrying current is called potential gradient.

Potentiometer can be used to compare emfs of two primary cells. The circuit diagram is shown below.



With the help of two way key we join cell ε_1 in the circuit and find out balancing length l_1. According to principle of potentiometer, fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross section and a constant current flows through it.

$\varepsilon_1 = Kl_1$ ... (i)

where K is potential gradient.

Similarly

$\varepsilon_2 = Kl_2$ ... (ii)

From equations (i) and (ii), we get $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{l_1}{l_2}$

22. Here, length of cubic domain, $l = 10^{-4}$ m

Volume of domain, $V = (10^{-4})^3 = 10^{-12}\ \text{m}^3 = 10^{-6}\ \text{cm}^3$

Mass of domain = Volume × density = $10^{-6} \times 7.9$ g

It is given that 55 g of iron contain 6.023×10^{23} iron atoms (Avogadro's number).

∴ Number of atoms in the domain

$$= \frac{6.023 \times 10^{23} \times 7.9 \times 10^{-6}}{55} = 8.65 \times 10^{16} \text{ atoms}$$

Maximum possible dipole moment is achieved when all the atomic dipole moments are perfectly aligned (which of course is unrealistic)

∴ $M_{max} = (8.65 \times 10^{16}) \times (9.27 \times 10^{-24})$
$= 8.0 \times 10^{-7}\ \text{A m}^2$

Maximum intensity of magnetisation

$$I_{max} = \frac{M_{max}}{\text{Volume of domain}} = \frac{8.0 \times 10^{-7}}{10^{-12}}$$

$= 8 \times 10^5\ \text{A m}^{-1}$

OR

Component of velocity of proton along x-axis will be $v_x = v \cos 30° = 8 \times 10^6 \times \dfrac{\sqrt{3}}{2}$
$= 6.93 \times 10^6\ \text{m s}^{-1}$

Component of velocity of proton along y-axis will be $v_y = v \sin 30° = 8 \times 10^6 \times \dfrac{1}{2} = 4 \times 10^6\ \text{m s}^{-1}$

Since the angle between velocity component v_x and magnetic field B is 0°, therefore magnetic force on proton due to component of velocity v_x will be $F = qv_x B \sin 0° = 0$.

Thus the proton will move uniformly along x-axis. As the component of velocity of proton along y-axis, v_y is perpendicular to the direction of magnetic field, therefore, the magnetic force on proton,

$F = qv_y B \sin 90° = q\, v_y B$ $\quad$ ($\because$ $\sin 90° = 1$)

will act as centripetal force and proton will describe a circular path due to this component of velocity. As the proton covers linear distance as well as describes a circular path, hence the path of the proton will be helix.

Radius of the helix will be given by

or $r = \dfrac{mv_y}{qB} = \dfrac{1.67 \times 10^{-27} \times 4 \times 10^6}{1.6 \times 10^{-19} \times 0.30}$

$= 13.92 \times 10^{-2}$ m = 13.92 cm

23. (i) Here, $n = 20$, $\varepsilon = 2$ V, $r = 0.5\ \Omega$, $R = 10\ \Omega$

If all the 20 cells are correctly connected in series to the external load of resistance R, then current in load is $I = \dfrac{n\varepsilon}{R + nr} = \dfrac{20 \times 2}{10 + 20 \times 0.5} = 2$ A

It means Sanju followed this set up as he succeded.

(ii) If one cell is wrongly connected in series arrangement of cells, it then reduces the total emf of the set up by an amount equals to two times the emf of each cell. Let m cells be connected wrongly by Shyam in series of total n cells, then he got the current 1.4 A in the external load of resistance R (= 10 Ω). Therefore,

$$I = 1.4 = \frac{(n-2m)\varepsilon}{(R+nr)} = \frac{(20-2m)\times 2}{10+20\times 0.5}$$

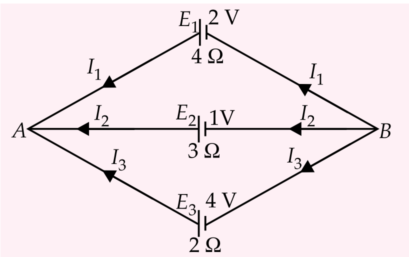
or $1.4 \times 20 = 40 - 4m$

$$\therefore \quad m = \frac{40-28}{4} = 3$$

It means that three cells are connected wrongly by Shyam.

(iii) Sanju has proper knowledge and he is careful in handling the apparatus. That is why, he got the required results from his setup. Shyam's knowledge is incomplete and his handling is careless. That is why he could not get the required results. Hence, to get proper results, one has to be extra careful and should plug all loopholes.

24. The scheme of connections is shown in figure. Let I_1, I_2 and I_3 be the currents flowing through the three cells E_1, E_2 and E_3.



Applying Kirchhoff's junction law at the junction A, we get, $I_1 + I_2 + I_3 = 0$

or $\quad I_3 = -(I_1 + I_2)$ ...(i)

Applying Kirchhoff's loop law to the closed loop BE_1AE_2B and we get, $4I_1 - 2 - 3I_2 + 1 = 0$

or $\quad 4I_1 - 3I_2 = 2 - 1 = 1$ ... (ii)

Applying the Kirchhoff's loop law to the closed loop BE_1AE_3B, we get

$4I_1 - 2 - 2I_3 + 4 = 0$

or $\quad 4I_1 - 2I_3 = 2 - 4 = -2$

or $\quad 4I_1 + 2 \times (I_1 + I_2) = -2$ (Using (i))

or $\quad 6I_1 + 2I_2 = -2$

or $\quad 3I_1 + I_2 = -1$ ... (iii)

Multiplying (iii) by 3 and adding to (ii), we get

$$(9+4)I_1 = 1 - 3 = -2 \quad \text{or} \quad I_1 = -\frac{2}{13}\text{A}$$

From (iii),

$$I_2 = -1 - 3I_1 = -1 - 3\left(-\frac{2}{13}\right) = -1 + \frac{6}{13} = -\frac{7}{13}\text{A}$$

From (i),

$$I_3 = -\left[-\frac{2}{13} - \frac{7}{13}\right] = \frac{9}{13}\text{A}$$

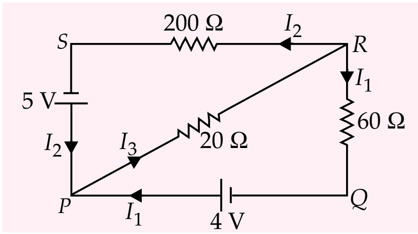
Note : Negative sign of currents shows that the actual direction is opposite to what has been taken in figure.

OR

Two rules which are used to find the current in different branches of electric circuit are:

(i) Kirchhoff's junction rule: It states that at any junction in an electrical circuit, sum of incoming currents is equal to sum of outgoing currents.

(ii) Kirchhoff's loop rule: It states that in any closed loop in a circuit, algebraic sum of applied emf's and potential drops across the resistors is equal to zero.



Applying Kirchhoff's loop rule to the closed loop $PRSP$, we get

$20I_3 + 200I_2 - 5 = 0$ ...(i)

Applying Kirchhoff's loop rule to the closed loop $PRQP$, we get

$20I_3 + 60I_1 - 4 = 0$ ...(ii)

Applying Kirchhoff's first rule to the junction P

$I_3 = I_1 + I_2$ ...(iii)

Substituting the value of I_3 in (i) and (ii) we get

$20I_1 + 220I_2 - 5 = 0$ ...(iv)

$80I_1 + 20I_2 - 4 = 0$ ...(v)

Solving equations (iv) and (v), we get

$$I_1 = \frac{39}{860}\text{ A}, \quad I_2 = \frac{16}{860}\text{ A} = \frac{4}{215}\text{ A}$$

From equation (iii),

$$I_3 = I_1 + I_2 = \frac{55}{860}\text{ A} = \frac{11}{172}\text{ A}$$

25. Refer point 3.1(1) page no. 169 and point 3.1 (3 (vi), (viii) Page no. 170 (MTG Excel in Physics).

OR

Refer point 3.8(8) page no. 180, 181 (MTG Excel in Physics).

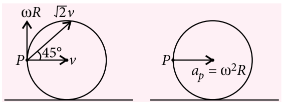
26. Refer point 3.3(5) page no. 173 (MTG Excel in Physics).

OR

Refer point 3.2(5, 6) page no. 171, 172 (MTG Excel in Physics).

PHYSICS MUSING

SOLUTION SET-23

1. **(b) :** 

For pure rolling, $v = \omega R \quad \therefore \quad v_p = \sqrt{2}v = \sqrt{2}\omega R$
Since v and ω are constant
$\therefore$ P has only centripetal acceleration.
Hence, angle between velocity and acceleration is 45°

2. **(b):** Total kinetic energy of ball in position $B = mg(R - r)$
Here, m = mass of ball.
Since, it rolls without slipping, the ratio of rotational to translational kinetic energy will be $\frac{2}{5}$

or $\frac{K_R}{K_T} = \frac{2}{5} \quad \therefore \quad K_T = \frac{5}{7}mg(R-r)$

or $\frac{1}{2}mv^2 = \frac{5}{7}mg\,(R-r)$ or $v = \sqrt{\frac{10g(R-r)}{7}}$

$\therefore \quad \frac{v}{R-r} = \sqrt{\frac{10g}{7(R-r)}}$

3. **(d) :** $\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$
Since direction of object velocity and image velocity is same, thus for image to be moving towards the lens, image should lie in object side *i.e.* $u < f$.

4. **(c):** From conservation of angular momentum, $I_0\omega_0 = I\omega$ where I_0 and I are the moments of inertia at temperature T_0 and T respectively.

Now $I_0 = \frac{2}{5}ma_0^2$ and $I = \frac{2}{5}ma^2$

Hence $\omega = \left(\frac{I_0}{I}\right)\omega_0 = \left(\frac{a_0}{a}\right)^2 \omega_0$

where a is radius of the sphere at T.

5. **(b) :** Terminal velocity is given by

$$v_T = \frac{2}{9}\frac{r^2(\rho_1 - \rho_2)g}{\eta}$$

$$(v_T)_A = \frac{2}{9}r^2\frac{(3.2-1)g}{\eta_w} \qquad \text{...(i)}$$

$$(v_T)_B = \frac{2}{9}r^2\frac{(6.0-1.6)g}{\eta_l} \qquad \text{...(ii)}$$

Given, $(v_T)_A = (v_T)_B$
Equating eqns (i) and (ii), we get $\frac{\eta_w}{\eta_l} = 0.5$

6. **(d) :** As $n_1\lambda_1 = n_2\lambda_2 \;\therefore\; n_2 = \frac{n_1\lambda_1}{\lambda_2} = 30 \times \frac{4200}{7000} = 18$

7. **(c):** $\frac{I_1}{I_2} = \frac{A^2}{(2A)^2} = \frac{1}{4}$ or $I_2 = 4I_1$ ...(i)

Now, $I_{max} = I_0 = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I_1} + \sqrt{4I_1})^2 = 9I_1$

or $I_1 = \frac{I_0}{9}$ ...(ii)

Now, $I_\phi = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= I_1 + 4I_1 + 2\sqrt{4I_1^2}\cos\phi$ (using (i))

$= 5I_1 + 4I_1\cos\phi = I_1(5 + 4\cos\phi)$

$= \frac{I_0}{9}(5 + 4\cos\phi)$ (using (ii))

8. As tension is changing, fundamental frequency

$f = \frac{1}{2L}\sqrt{\frac{T}{m}} = k\sqrt{T}$ with $k = \frac{1}{2L\sqrt{m}}$

$\therefore \frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T},\; \frac{\Delta T}{T} = \frac{2\Delta f}{f} = 2 \times \frac{5}{500} \times 100\% = 2\%$

9. Fundamental frequency of an organ pipe, $f = \frac{v}{2l}$
where l is length of the organ pipe at temperature T
Also, $l = l_0[1 + \alpha(T - T_0)]$

Velocity of sound, $v = \sqrt{\frac{\gamma RT}{M}}$

We have to find the temperature at which $f(T) = f(T_0)$ for small $(T - T_0)$

$$\frac{\sqrt{\frac{\gamma RT}{M}}}{2l_0[1+\alpha(T-T_0)]} = \frac{\sqrt{\frac{\gamma RT_0}{M}}}{2l_0} \Rightarrow \sqrt{\frac{T}{T_0}} = 1 + \alpha(T - T_0)$$

$$\left[1 + \left(\frac{T-T_0}{T_0}\right)\right]^{1/2} = 1 + \alpha(T - T_0)$$

For small $(T–T_0)$ we may use binomial approximation,

$$1 + \frac{1}{2}\left(\frac{T-T_0}{T_0}\right) = 1 + \alpha(T - T_0)$$

$$\frac{1}{2}\left(\frac{T-T_0}{T_0}\right) = \alpha(T - T_0),\; \alpha = \frac{1}{2T_0} \text{ or } T_0 = \frac{1}{2\alpha}$$

10. Wave velocity on a string is given by $v = \sqrt{T/\mu}$.
The tension in the string is variable, being minimum at point B and maximum at A. Therefore the wave velocity varies. The frequency of a wave is unchanged throughout the propagation, as long as the source vibrates with constant frequency.

$\therefore \quad v_A = \upsilon\lambda_A$ and $v_B = \upsilon\lambda_B$

$$\frac{v_A}{v_B} = \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{T_A}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

$\therefore \quad \lambda_A = 2\lambda_B = 2 \times 0.06 = 0.12$ m

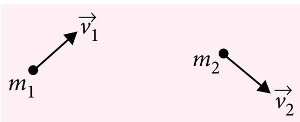
CORE CONCEPT on

Analysis of System of Two Masses in COM Frame

There are numerous questions involving two masses which when observed from ground frame are either a little dif cult to solve or lengthy in calculations.

A better option in such cases would be if we try to analyse the motion of the system from centre of mass (COM) frame.

Let us begin with a situation where we have two masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ with respect to ground frame.



Hence, the velocity of COM becomes

$$\vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$\therefore$ The velocities of the masses m_1 and m_2 in the frame of COM becomes $\vec{v}_{1c}$ and $\vec{v}_{2c}$ where

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_c = \vec{v}_1 - \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}\right)$$

$$= \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2}$$

$$\therefore \ \vec{v}_{1c} = \left(\frac{m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2)$$

Similarly, $\vec{v}_{2c} = \left(\dfrac{m_1}{m_1 + m_2}\right)(\vec{v}_2 - \vec{v}_1)$

These equations just looks like any ordinary equation but a close introspection shows a beautiful result as below:

The linear momentum of the masses m_1 and m_2 in COM frame becomes,

$$\vec{p}_{1c} = m_1\vec{v}_{1c} = \left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2)$$

$$\vec{p}_{1c} = \mu(\vec{v}_1 - \vec{v}_2)$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system

Similarly, $\vec{p}_{2c} = \mu(\vec{v}_2 - \vec{v}_1)$

Clearly note that $\vec{p}_{1c} = -\vec{p}_{2c}$, which means irrespective of the values of $\vec{v}_1$ and $\vec{v}_2$, whatever be their direction of motion, the magnitude of linear momentum of the masses with respect to centre of mass is equal in magnitude and opposite in direction.

Also, $\quad p_{1c} = p_{2c} = \mu v_{rel}$

where $\quad v_{rel} = |\vec{v}_1 - \vec{v}_2|$

$\therefore$ The kinetic energy of the masses in COM frame

$$K_{1c} = \frac{p_{1c}^2}{2m_1}$$

$$K_{2c} = \frac{p_{2c}^2}{2m_2}$$

$\therefore$ Kinetic energy of the system in its COM frame becomes

$$K_{cf} = K_{1c} + K_{2c}$$

$$= \frac{1}{2}\left(\frac{p_{1c}^2}{m_1} + \frac{p_{2c}^2}{m_2}\right)$$

but $p_{1c} = p_{2c} = \mu v_{rel}$

$$\therefore \ K_{cf} = \frac{1}{2}\mu v_{rel}^2$$

Now, let us try to correlate the relation between kinetic energy in ground frame of system (K_{gf}) and kinetic energy in COM frame of system (K_{cf})

$$K_{gf} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}m_1(\vec{v}_{1c} + \vec{v}_c)^2 + \frac{1}{2}m_2(\vec{v}_{2c} + \vec{v}_c)^2$$

$$= \frac{1}{2}m_1(v_{1c}^2 + v_c^2 + 2\vec{v}_{1c}\cdot\vec{v}_c) + \frac{1}{2}m_2 (v_{2c}^2 + v_c^2 + 2\vec{v}_{2c}\cdot\vec{v}_c)$$

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

$$\therefore \quad K_{gf} = \left(\frac{1}{2}m_1v_{1c}^2 + \frac{1}{2}m_2v_{2c}^2\right) + \left(\frac{1}{2}(m_1+m_2)v_c^2\right) + \frac{1}{2}(2m_1\vec{v}_{1c}\cdot\vec{v}_c + 2m_2\vec{v}_{2c}\cdot\vec{v}_c)$$

where, $\frac{1}{2}m_1v_{1c}^2 + \frac{1}{2}m_2v_{2c}^2 = K_{cf}$

$$\frac{1}{2}(m_1+m_2)v_c^2 = \text{Kinetic energy of COM} = K_{COM}$$

$$m_1\vec{v}_{1c}\cdot\vec{v}_c + m_2\vec{v}_{2c}\cdot\vec{v}_c = (m_1\vec{v}_{1c} + m_2\vec{v}_{2c})\cdot\vec{v}_c = 0$$

[since linear momentum of system in COM frame would be zero]

$\therefore \quad K_{gf} = K_{cf} + K_{COM}$

$$= \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}(m_1+m_2)v_c^2$$

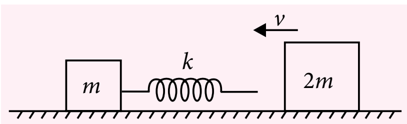
where v_c = velocity of COM in ground frame.

If instead of ground frame, we choose any other frame K_{COM} would represent the kinetic energy of COM in that frame.

Hence clearly, the COM frame is the frame of least kinetic energy.

Remember these results. They will help us in solving several questions, some of which have been shown below:

1. On a frictionless surface a block of mass $2m$ is projected towards an unstretched spring connected to a block of mass m. Find maximum compression in spring.



Soln.: Method-1 :

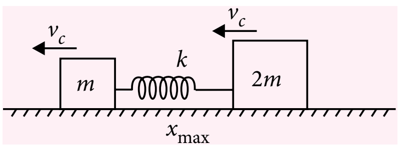
In ground frame, on $(m + 2m)$ system, $F_{ext} = 0$

$\therefore \quad \Delta p = 0$

i.e., no change in linear momentum of system.

As soon as the $2m$ strikes spring, its velocity starts decreasing whereas of m starts increasing, due to which deformation starts increasing initially and reaches maximum till $2m$ travels faster than m.

Hence at maximum compression, both attain same velocity.



$\therefore \quad \Delta p = 0$

$\Rightarrow \quad 2mv = (m + 2m)v_c$

$$v_c = \frac{2}{3}v \qquad \text{... (i)}$$

Using work energy theorem

$W_{spring} = \Delta K$

$\Rightarrow \; -\Delta U = \Delta K$

$$\Rightarrow \; -\left[\frac{1}{2}kx_{max}^2 - 0\right] = \frac{1}{2}(m+2m)v_c^2 - \frac{1}{2}2mv^2$$

$$\Rightarrow \; x_{max} = \sqrt{\frac{2}{3}\frac{m}{k}}\,v \qquad \text{(Using (i))}$$

Method-2 :

In COM frame,

$W_{spring} = \Delta K$

$$\Rightarrow \; -\Delta U = (K_{cf})_f - (K_{cf})_i$$

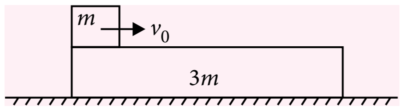
Since, $(K_{cf})_f$ = final KE in COM frame

$= 0 \quad [\because v_{rel} = 0]$

$$\therefore \quad -\left[\frac{1}{2}kx_{max}^2 - 0\right] = -\frac{1}{2}\left(\frac{m\cdot 2m}{3m}\right)v^2$$

$$\Rightarrow \; x_{max} = \sqrt{\frac{2}{3}\frac{m}{k}}\,v.$$

2. A mass m is projected over a rough long plank of mass $3m$ kept on a smooth horizontal surface as shown.



Find the work done by friction till relative slipping stops.

Soln.: In COM frame,

$$K_i = \frac{1}{2}\left(\frac{m\cdot 3m}{4m}\right)v_0^2 = \frac{1}{2}\left(\frac{3m}{4}\right)v_0^2$$

$K_f = 0 \qquad [\because v_{rel} = 0]$

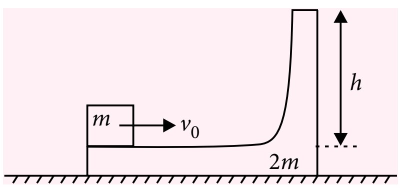
$\therefore$ Applying work energy theorem,

$W_{friction} = \Delta K$

$$= 0 - \frac{1}{2}\left(\frac{3m}{4}\right)v_0^2$$

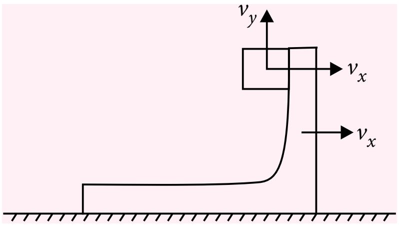
$$= -\frac{3}{8}mv_0^2$$

3.



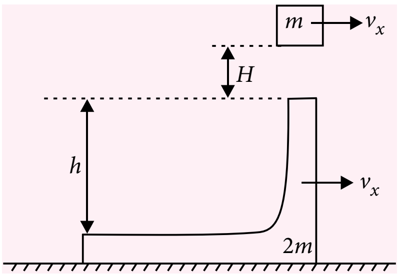
All surfaces are smooth. Mass m is projected over $2m$ whose other end is almost vertical of total height h. The mass m is found to go to a height H above the top edge of mass $2m$. Find H.

Soln.: When the block m is about to leave $2m$, the situation will look like as shown in figure:



The horizontal component of velocity of m will be same as the velocity of $2m$, and thereafter, after leaving contact, its vertical component would change leaving horizontal component of both identical.

Hence at maximum height,



$\therefore$ In COM frame, $K_f = 0$ $\qquad [\because v_{rel} = 0]$

$\therefore$ Applying work energy theorem,

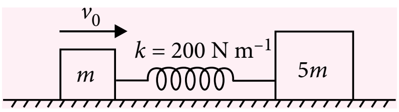
$W_g = \Delta K$

$$\Rightarrow -\Delta U_g = (K_{cf})_f - (K_{cf})_i$$

$$\Rightarrow -[mg(h+H)] = -\frac{1}{2}\frac{2m}{3}\cdot v_0^2$$

$$\therefore H = \frac{v_0^2}{3g} - h$$

4. Two blocks are connected by an unstretched spring and mass m is given a velocity $v_0 = 24$ m s^{-1} towards $5m$ as shown.



Find the maximum velocity of $5m$?

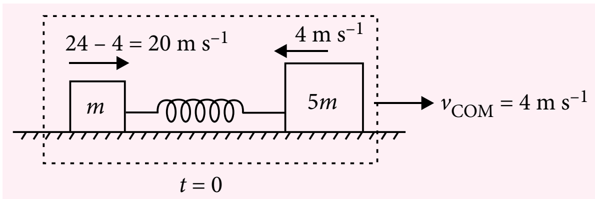
Soln.: This is an example where the two masses oscillate about their COM frame (which moves with a constant velocity) with a time period,

$$T = 2\pi\sqrt{\frac{\mu}{k}} = 2\pi\sqrt{\frac{5m}{6k}}$$

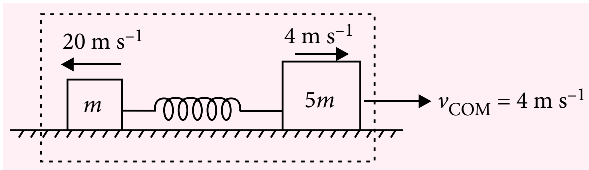
$\therefore$ Velocity of COM,

$$v_{COM} = \frac{mv_0}{m+5m} = \frac{v_0}{6} = 4 \text{ m s}^{-1}$$

$\therefore$ With respect to centre of mass, the given situation looks as shown in figure below:



But after half oscillation, *i.e.* $t = \frac{T}{2}$ later



The velocity of any oscillating system is maximum at mean position only and here obviously the mean position is unstretched length.

But, $\vec{v}_{5m} = \vec{v}_{5mc} + \vec{v}_{COM}$

where $\vec{v}_{5m}$ = velocity of $5m$ in ground frame

$\vec{v}_{5mc}$ = velocity of $5m$ in COM frame

$\vec{v}_{COM}$ = velocity of COM

= 4 m s^{-1} towards right

Now, if $\vec{A} = \vec{B} + \vec{C}$, then A is maximum only if $\vec{B} \| \vec{C}$, hence

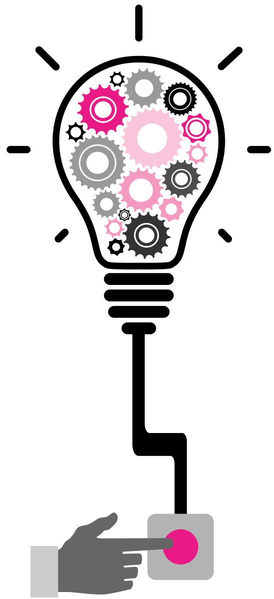
$|\vec{v}_{5m}|_{max}$ only if $\vec{v}_{5mc} \| \vec{v}_{COM}$

$\therefore$ $(v_{5m})_{max} = (v_{5mc})_{max} + v_{COM}$
$= 4 + 4 = 8$ m s^{-1}.

"There is no science in this world like physics. Nothing comes close to the precision with which physics enables you to understand the world around you. It's the laws of physics that allow us to say exactly what time the sun is going to rise. What time the eclipse is going to begin. What time the eclipse is going to end."

–Neil deGrasse Tyson

THOUGHT PROVOKING PROBLEMS



By : Prof. Rajinder Singh Randhawa*

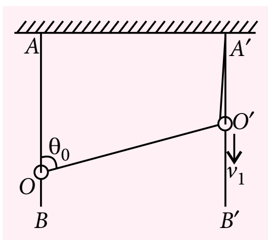
KINEMATICS

1. A particle is projected so as to have a range R on the horizontal plane through the point of projection. If θ_1, θ_2 are the possible angles of projection and t_1, t_2 the corresponding times of flight, prove that
$$\frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}.$$

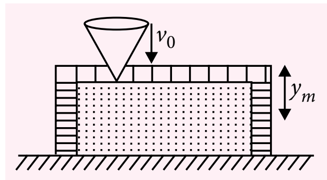
2. A body of mass m is projected in a resisting medium whose resistive force is $F = kv$ and the initial velocity is u. Find
   (a) the expression for position and velocity in terms of time.
   (b) the time after which the velocity becomes $\frac{u}{2}$.
   (c) the time taken for the body to come to rest.
   (d) the total distance covered by the body.

3. A rubber ball is to be thrown from ground over a vertical wall (of height H) from a distance d. (a) At what minimum initial velocity v_0 is this possible? (b) At what angle θ to the horizontal should the velocity be directed in this case?

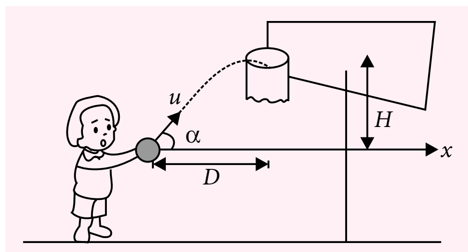
4. Two rings O and O' are put on two vertical stationary rods AB and $A'B'$ respectively. An inextensible string is fixed at point A' and on ring O and is passed through ring O'. Assuming that ring O' moves downwards at a constant velocity v_1, find the velocity v_2 of ring O if $\angle AOO' = \theta_0$.



5. A steel cone falls on a wooden block. It penetrates to a maximum depth y_m in the wood.
The acceleration of the cone after impact is $a = g - cy^2$ where c is constant and y is penetration depth. If initial velocity is v_0, find the constant c.



6. A basketball player throws the ball with initial velocity u at an angle α with the horizontal to the hoop which is located at a horizontal distance D and at a height H above the point of release, as shown in the figure.
(a) Show that the initial speed required is given by
$$u = \sqrt{\frac{gD}{2\cos^2\alpha\left(\tan\alpha - \frac{H}{D}\right)}}.$$
(b) Show that the angle β to the horizontal at which it reaches the hoop is given by $\tan\beta = \frac{2H}{D} - \tan\alpha$



***Randhawa Institute of Physics,** S.C.O. 208, First Fl., Sector-36D & S.C.O. 38, Second Fl., Sector-20C, Chandigarh, Ph. 09814527699

SOLUTIONS

1. Since, the range R is same for angles of projection θ_1 and θ_2, we have $\theta_1 + \theta_2 = 90°$

Now, $t_1 = \dfrac{2u\sin\theta_1}{g}$ and $t_2 = \dfrac{2u\sin\theta_2}{g}$

$$\Rightarrow \frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\left(\frac{2u}{g}\sin\theta_1\right)^2 - \left(\frac{2u}{g}\sin\theta_2\right)^2}{\left(\frac{2u}{g}\sin\theta_1\right)^2 + \left(\frac{2u}{g}\sin\theta_2\right)^2}$$

$$= \frac{\sin^2\theta_1 - \sin^2\theta_2}{\sin^2\theta_1 + \sin^2\theta_2}$$

$$= \frac{\sin(\theta_1 - \theta_2)\sin(\theta_1 + \theta_2)}{\sin^2\theta_1 + \sin^2(90° - \theta_1)} = \frac{\sin(\theta_1 - \theta_2)\sin 90°}{\sin^2\theta_1 + \cos^2\theta_1}$$

$$\therefore \frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \quad \left(\begin{array}{l}\because \sin^2\theta_1 + \cos^2\theta_1 = 1 \\ = \sin 90° = \sin(\theta_1 + \theta_2)\end{array}\right)$$

2. (a) Here, $a = \dfrac{dv}{dt} = -\dfrac{k}{m}v \Rightarrow \displaystyle\int_u^v \frac{dv}{v} = -\int_0^t \frac{k}{m}dt$

$$\Rightarrow \ln\frac{v}{u} = -\frac{k}{m}t \text{ or } v = ue^{-kt/m}$$

Also, $\dfrac{dx}{dt} = ue^{-kt/m} \Rightarrow \displaystyle\int_0^x dx = \int_0^t ue^{-kt/m}\,dt$

$$\therefore x = \left[\frac{u\,e^{-kt/m}}{-k/m}\right]_0^t = -\frac{mu}{k}(e^{-kt/m} - 1)$$

(b) If t' = time after which velocity becomes $u/2$ then,

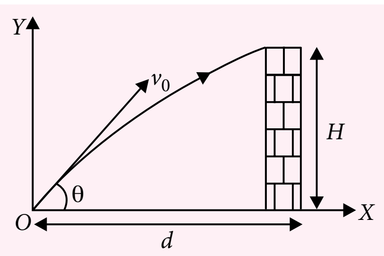
$$\frac{u}{2} = ue^{-kt'/m} \Rightarrow e^{kt'/m} = 2$$

Solving, we get, $t' = \dfrac{m}{k}\ln 2$

(c) Since $v = u\,e^{-kt/m}$, the body shall come to rest when $t \to \infty$.

(d) When $t \to \infty$, $x = mu/k$. As the body moves throughout in one direction only, the total distance covered by the body is $\dfrac{mu}{k}$.

3.



From figure,

$$d = v_0\cos\theta\cdot t \quad \text{...(i)}$$

$$H = v_0\sin\theta\cdot t - \frac{1}{2}gt^2 \quad \text{...(ii)}$$

Since number of variables is three (v_0, θ and t), so we require three equations. Third equation comes from the condition that v_0 is minimum,

$$\frac{dv_0}{d\theta} = 0 \quad \text{...(iii)}$$

Now, from (i) and (ii), eliminating t we get,

$$H = v_0\sin\theta \times \frac{d}{v_0\cos\theta} - \frac{1}{2}g\left(\frac{d}{v_0\cos\theta}\right)^2$$

$$\therefore H = d\tan\theta - \frac{gd^2}{2v_0^2}\sec^2\theta \quad \text{...(iv)}$$

Differentiating equation (iv) w.r.t. θ, we get

$$0 = d\sec^2\theta - \frac{gd^2}{2}\left[\frac{-2}{v_0^3}\sec^2\theta\frac{dv_0}{d\theta} + \frac{1}{v_0^2}\times 2\sec^2\theta\tan\theta\right]$$

Putting $\dfrac{dv_0}{d\theta} = 0$, we get

$$\frac{gd^2}{2}\left[\frac{2}{v_{\min}^2}\sec^2\theta\tan\theta\right] = d\sec^2\theta$$

$$\Rightarrow v_{\min}^2 = gd\tan\theta \quad \text{...(v)}$$

From eqns. (iv) and (v), we get

$$H = d\tan\theta - \frac{gd^2\sec^2\theta}{2gd\tan\theta}$$

Solving, we get, $d\tan^2\theta - 2H\tan\theta - d = 0$

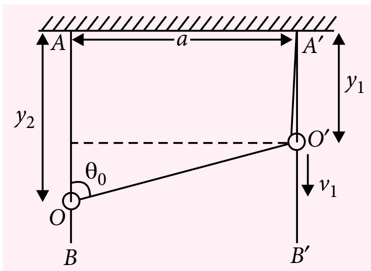
$$\Rightarrow \tan\theta = \frac{2H + \sqrt{4H^2 + 4d^2}}{2d}$$

$$\therefore \theta = \tan^{-1}\left[\frac{H + \sqrt{H^2 + d^2}}{d}\right]$$

From eqn. (v), $v_{\min} = \sqrt{g(H + \sqrt{H^2 + d^2})}$

4. Since the length of string is constant,

so, $y_1 + \sqrt{(y_2 - y_1)^2 + a^2} = \text{constant}.$



Differentiating with respect to time, we get

$$\frac{dy_1}{dt} + \frac{1}{2}\frac{2(y_2 - y_1)}{\sqrt{(y_2 - y_1)^2 + a^2}}\left(\frac{dy_2}{dt} - \frac{dy_1}{dt}\right) = 0$$

or $\quad v_1 + (v_2 - v_1)\cos\theta = 0$

$\Rightarrow \quad v_2 \cos\theta + v_1 (1 - \cos\theta) = 0$

or $\quad v_2 = - v_1 (1 - \cos\theta)/\cos\theta$

At given instant $\theta = \theta_0$

$v_2 = - v_1 (1 - \cos\theta_0)/\cos\theta_0$

Negative sign shows that ring O moves upward with speed v_2 at the given instant.

5. Acceleration, $a = g - cy^2 \Rightarrow v\dfrac{dv}{dy} = g - cy^2$

Separating variables, $vdv = (g - cy^2)dy$

Integrating both sides, we get,

$$\int_{v_0}^{0} v\,dv = \int_{0}^{y_m} (g - cy^2)dy$$

$$-\frac{v_0^2}{2} = gy_m - \frac{cy_m^3}{3} \text{ or } c = \frac{6gy_m + 3v_0^2}{2y_m^3}$$

6. (a) Using equation of trajectory

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

We get $H = D\tan\alpha - \dfrac{gD^2}{2u^2\cos^2\alpha}$

$$\Rightarrow \frac{gD^2}{2u^2\cos^2\alpha} = D\tan\alpha - H$$

$$\Rightarrow u^2 = \frac{gD^2}{2\cos^2\alpha\,(D\tan\alpha - H)}$$

$$\therefore u = \sqrt{\frac{gD}{2\cos^2\alpha\left(\tan\alpha - \frac{H}{D}\right)}}$$

(b) When the ball reaches the hoop, its horizontal and vertical components of velocity are given by

$$v_x = u\cos\alpha \text{ and } v_y^2 - (u\sin\alpha)^2 = 2(-g)H$$

$$\Rightarrow v_y = \sqrt{u^2\sin^2\alpha - 2gH}$$

$$\Rightarrow \tan\beta = \frac{v_y}{v_x} = \frac{\sqrt{u^2\sin^2\alpha - 2gH}}{u\cos\alpha}$$

Putting the value of u, we get,

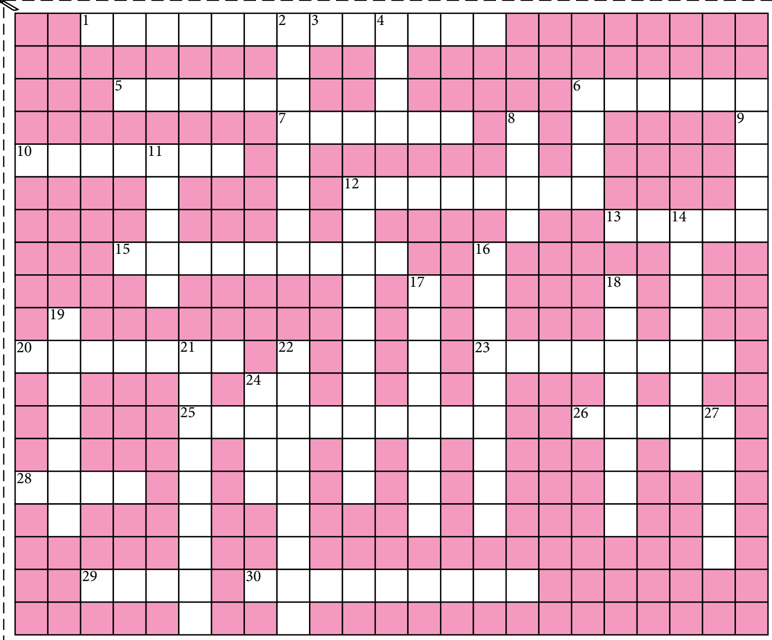
$$\tan\beta = \sqrt{\tan^2\alpha - 4H\left[\frac{\tan\alpha}{D} - \frac{H}{D^2}\right]}$$

$$\Rightarrow \tan\beta = \sqrt{\left(\frac{2H}{D} - \tan\alpha\right)^2} \quad \therefore \tan\beta = \frac{2H}{D} - \tan\alpha$$

CROSS WORD

Readers can send their answer with complete address before 15th of every month to win exciting prizes. Winners' name with their valuable feedback will be published in next issue.

Cut Here



ACROSS

1. An accelerator in which two beams travelling in opposite directions are steered together to provide high-energy collisions between the particles in one beam and those in the other. (8)
3. A radioactive decay product of uranium often found in uranium ore. (6)
5. A magma body that has solidified at depth in the Earth's crust. (6)
6. A property of some elementary particles that causes them to exert forces on one another. (6)
7. The front surface of a TV, or other cathode-ray tube, suitably coated, on which the visible pattern is displayed. (6)
10. An instrument for measuring angles (up to 120°) between two objects and particularly angle between an astronomical body and the horizon. (7)
12. A type of cold cathode scaling tube in which the cathodes are shaped into the form of characters usually the digits 0 to 9. (8)
13. A figure traced out by the luminous spot on the screen of a cathode ray tube. (5)
15. A direct electric current, especially one produced by chemical action. (9)
20. The CGS electromagnetic unit of magnetomotive force or magnetic potential. (7)
23. The speed of a wave deduced from tracking individual wave crests. (8)
25. The science of rotating mechanical devices. (10)
26. Transient air glow events observed near 90 km, nearly simultaneously with a strong cloud-to-ground lightning stroke. (5)
28. Differentiated central volume of the Earth and some other planets. (4)
29. Absence of electron in a semiconductor. (4)
30. The study of measuring the power of lens. (9)

DOWN

2. Energy in one mole of photons equivalent to 6.023×10^{23} photons. (8)
4. A CGS unit of force. (4)
6. The European Organization for Nuclear Research, the world largest particle physics centre. (4)
8. A unit prefix in the matric system denoting multiplication by 10^{12}. (4)
9. A steep wave that moves up narrowing channels produced either by regular tidal events, or as a result of tsunami. (4)
11. Moon of Saturn discovered by R. Terrile in 1980 in voyager photos. (5)
12. The removal of adsorbed gas from a solid surface during which process, heat is taken from the surface. (10)
14. Moon of Jupiter discovered by E. Barnard in 1892. (8)
16. Degree of exactness in a measurement. (9)
17. Fermi National Accelerator Laboratory in Batavia, named for particle physics pioneer Enrico Fermi. (8)
18. The study of fire and heat especially with regard to chemical analysis. (8)
19. Widely distributed flux predominately from OH, oxygen and neon at an altitude of 85 to 95 km. (7)
21. An instrument that measures the rate of flow of fluids. (9)
22. The infall of matter onto a body, such as a planet, a forming star, or a black hole, occurring because of their mutual gravitational attraction. (9)
24. A general term denoting the quantity of radiation or energy absorbed in a specific mass. (4)
27. A visible disruptive discharge of electricity between two places at opposite high potential. (5)